

the radius opposite the pole. Various applications are given, including the determination of the sharp bound in Hadamard's three circles theorem. That is, we suppose that $f(z)$ is regular and single-valued for $q \leq |z| \leq 1$, that $|f(z)| \leq p$ for $|z| = q$ and $|f(z)| \leq 1$ for $|z| = 1$, and find the largest possible value for $|f(z_0)|$, where z_0 is some point within the ring. A formula for the bound is given in terms of theta functions, and the problem is also discussed geometrically. In particular, if $q < p < 1$, then the maximum value of $|f(z_0)|$ is attained by a function $f(z)$ which is univalent in $q < |z| < 1$, and maps this ring on $|w| < 1$ excluding an arc of $|w| = p$. (Received January 23, 1943.)

114. Raphael Salem: *Sets of uniqueness and sets of multiplicity.*

An algebraic integer α having the property that all its conjugates have their moduli inferior to 1 will be called a "Pisot number" (α is necessarily real and greater than 1). The following theorems are proved: I. Let $0 < \xi < 1$. If the Fourier-Stieltjes transform $\sum_{k=0}^{\infty} \cos \pi u \xi^k$ does not tend to zero for $u \rightarrow \infty$, then $1/\xi$ is a Pisot number. II. Let $0 < \xi < 1/2$, and let P be the symmetrical perfect set of Cantor type and of constant ratio of dissection ξ constructed on $(0, 2\pi)$ (relative length of the black intervals is $1 - 2\xi$). Then P is a set of uniqueness for trigonometrical series if (and only if) $1/\xi$ is a Pisot number. III. There exist Pisot numbers of the form $2 + \epsilon$, ϵ being positive and arbitrarily small; hence, there exist sets of uniqueness which are of Hausdorff dimensionality as near to 1 as desired. (Received January 11, 1943.)

115. Gabor Szegő: *On the oscillation of differential transforms. IV. Jacobi polynomials.*

Let $\alpha \geq 0$, $\beta \geq 0$, $c \geq 0$. In a recent paper (Trans. Amer. Math. Soc. vol. 52 (1942) pp. 463-497, cf. p. 489), E. Hille proved the following two theorems: (A) The differential operator $\vartheta - c = (1 - x^2)D^2 + [\beta - \alpha - (\alpha + \beta + 2)x]D - c$, $D = d/dx$, does not diminish the number of the sign changes of a function in $-1 < x < +1$; (B) If the number of the sign changes of $(\vartheta - c)^k f(x)$ remains less than or equal to N for all k , $k = 1, 2, 3, \dots$, then $f(x)$ is a polynomial of degree less than or equal to N . The purpose of the present note is the extension of Theorem A to $\alpha > -1$, $\beta > -1$ and of Theorem B to arbitrary real values of α and β , in the latter case with the modification that the possible degree of the polynomial $f(x)$ is less than or equal to $N + \gamma$, $\gamma = \gamma(\alpha, \beta, c)$. (Received January 20, 1943.)

APPLIED MATHEMATICS

116. Stefan Bergman: *A formula for the stream function in compressible fluid flow.*

Using the hodograph method and a general representation for the stream function of a flow of an incompressible fluid (see Bergman, *Hodograph method in the theory of compressible fluid*, Publication of Brown University, 1942) the author gives an explicit formula for the stream functions of flows of certain types. (Received January 27, 1943.)

117. Nathaniel Coburn: *Boundary value problems in plane plasticity.* Preliminary report.

The following problem is discussed in this paper: given an infinite plate of perfectly plastic material bounded by the x -axis; to determine the stresses within the plate when the stresses on the boundary are known. First, the equation of plasticity (yield

condition) is linearized. It is shown that, in this case, the above problem reduces to finding a solution $F(x, y)$ for the hyperbolic equation under given Cauchy data. Next, it is shown that for certain types of boundary conditions, the stresses corresponding to the linear problem are equal to the stresses corresponding to the original nonlinear problem over a series of equally spaced lines parallel to the boundary (x -axis). The situation is analogous to two surfaces which do not coincide but do intersect in curves whose projections on the xy -plane are parallel lines. The method may be modified to give approximate solutions of the nonlinear problem throughout the plate. Some examples are worked. An indication is given of how the method may be applied to the finite rectangular plate. (Received January 2, 1943.)

118. H. J. Greenberg: *Application of a summability method in solving boundary value problems.*

In order to apply the method of particular solutions for solving boundary value problems for the elliptic linear partial differential equation $L(U)=0$, S. Bergman (Duke Math. J. vol. 6 (1940) p. 541) has introduced the complete set $P_{2n-\alpha}(z)$ of such solutions, by means of which every function U , $L(U)=0$, regular in the circle $x^2+y^2 < R^2$, $R > 0$, can be developed in the series $S: U(z) = \sum a_k P_k(re^{i\theta})$. By studying the "associated function" $\sum a_k \Gamma(1/2) \cdot \Gamma(k+1/2) z^k / \Gamma(k+1)$ necessary and sufficient conditions are given for the series S to be convergent on $|z|=R$ and, what is more important for applications, for S to be $(C, 1)$ summable on this circle of convergence. Conditions are given in each case under which the values thus obtained are the boundary values of the function $U(z)$. Using these results a method is given for the actual solution of boundary value and characteristic value problems for the equation $L(U)=0$. (Received January 29, 1943.)

GEOMETRY

119. John DeCicco: *Extensions of certain dynamical theorems of Halphen and Kasner.*

The theorem of Halphen which characterizes central fields of force and the general theorem of Kasner concerning one-third the curvatures is extended to generalized fields of force in space, which depend upon the position of the point and direction. The number of generalized fields of force whose dynamical trajectories are all plane curves is $\infty^{f(2)+f(4)+2f(6)}$. The ∞^5 generalized trajectories consist of ∞^2 systems of ∞^3 generalized plane trajectories, each system lying in a plane tangent to a given surface Σ . In an arbitrary positional field of force, Kasner showed that the rest trajectory and line of force through a given point O have the same osculating plane and that the ratio ρ of the curvature of the rest trajectory to that of the line of force is $1/3$. For generalized fields of force this theorem is no longer valid. All generalized fields of force such that the rest trajectory and the line of force through any point O of the space have the same osculating plane are determined; and, also in this class, the subclass of all generalized fields of force for which $\rho=1/3$. (Received January 2, 1943.)

120. Edward Kasner and John DeCicco: *Generalized dynamical trajectories in space.*

The differential geometry of positional fields of force has been developed in *Differential-geometric aspects of dynamics*, Amer. Math. Soc. Colloquium Publications vol. 3, 1913. In abstract 48-11-329, the authors began the study of generalized fields