### 105. L. L. Dines: *On linear combinations of quadratic forms.*

The author considers conditions under which $m$ given quadratic forms in $n$ variables admit a linear combination which is (1) definite, or (2) semi-definite. The paper will appear in full in an early issue of Bull. Amer. Math. Soc. (Received December 10, 1942.)

### 106. H. Schwerdtfeger: *Identities between skew-symmetric matrices.*

Let $P$, $Q$ be two $2m$-rowed skew-symmetric matrices, $P$ regular. Put $P^{-1}Q=A$. The determinant $|\lambda P-Q|$ equals $\kappa(\lambda)^2$ with $\kappa(\lambda)=k_0\lambda^m-k_1\lambda^{m-1}+\cdots+(-1)^m k_m$ where $k_0$, $k_m$ are the pfaffian parameters of $P$, $Q$, respectively, and $k_1, \cdots, k_{m-1}$ the rational simultaneous invariants of $P$ and $Q$. By Cayley's identity one has $\kappa(A)^2=(0)$. By means of known theorems (cf. for example, MacDuffee's *Theory of matrices*, Theorems 32.2, 32.3, and 29.3, or A. A. Bennett, Bull. Amer. Math. Soc. vol. 25 (1919) pp. 455–458) it follows that $\kappa(\lambda)$ has as a factor the highest invariant factor $h(\lambda)$ of $\lambda P-Q$, and thus the minimum polynomial of $A$. Hence follows $h(A)=(0)$ and $\kappa(A)=(0)$. This identity involving the skew-symmetric matrices $P$, $Q$ is of geometric interest; if $m=2$ one has, for instance: $k_0QP^{-1}Q=k_1Q-k_2P$ whence the elementary theory of a pair of null systems (linear complexes) in projective 3-space can be derived. (Received January 8, 1943.)

## Analysis

### 107. G. E. Albert: *An extension of Korous' inequality for orthonormal polynomials.*

Let $\{q_n(x)\}$ denote the set of polynomials orthonormal on $(a, b)$ with weight functions $p(x)r(x)$ where $0\leqq p(x)$ and $0\leqq r(x)\leqq M$. If a non-negative polynomial $\pi_m(x)$ of degree $m$ can be found such that the quotient $\pi_m(x)/r(x)$ satisfies a Lipschitz condition on $(a, b)$ and if $\{p_n(x)\}$ denotes the set of polynomials orthonormal on $(a, b)$ with weight function $p(x)[\pi_m(x)]^2$ then if the polynomials $\{p_n(x)\}$ are bounded uniformly with respect to $n$ and $x$ on any subset of $(a, b)$ the same is true of the set $\{q_n(x)\}$. This result follows from an inequality that is established by the same procedure as that used on an equiconvergence theorem by L. H. Miller and the author (abstract 49-3-108). If $r(x)$ is bounded from zero and satisfies a Lipschitz condition on $(a, b)$, the inequality mentioned reduces essentially to an inequality due to Korous (G. Szegö, *Orthogonal polynomials*, Amer. Math. Soc. Colloquium Publications vol. 23, 1939, p. 157). (Received January 19, 1943.)

### 108. G. E. Albert and L. H. Miller: *Equiconvergence of series of orthonormal polynomials.* Preliminary report.

Walsh and Wiener (Journal of Mathematics and Physics vol. 1 (1922)) found necessary and sufficient conditions for the equiconvergence of the expansions of an arbitrary function in terms of different systems of functions orthonormal on a finite interval. In the present paper these conditions are applied to the study of polynomials orthonormal relative to weight functions satisfying a variety of hypotheses. A remarkably simple proof is obtained for an equiconvergence theorem that includes one published by Szegö (*Orthogonal polynomials*, Amer. Math. Soc. Colloquium Publication, vol. 23, 1938, Theorem 13, 1.2) and the results given by Peebles (Proc. Nat. Acad. Sci. U.S.A. vol. 25 (1939) pp. 97–104). The application of the Walsh-Wiener conditions is based upon the observation that if $K_n^{(1)}(x, t)$ and $K_n^{(2)}(x, t)$ are the respective kernel polynomials for the systems orthonormal with respect to the weight functions $p(t)r_1(t)$ and $p(t)r_2(t)$, then (subject to integrability conditions) for arbitrary fixed $x$ the integral $\int_{-1}^{1}p(t)r_2(t)[r_1(t)r_2^{-1}(t)K_n^{(1)}(x, t)-K_n^{(2)}(x, t)]^2dt$ is the least squares integral of order $n$ for the function $r_1(t)r^{-1}(t)K_n^{(1)}(x, t)$ with respect to the system of weight $p(t)r_2(t)$. An expedient choice of a polynomial of degree $n$ to replace $K_n^{(2)}(x, t)$ and study of the result completes the proof. No asymptotic formulas of any kind are needed. (Received December 9, 1942.)

### 109. E. F. Beckenbach: *On conjugate harmonic functions.*

According to N. Cioranesco (*Sur les fonctions harmoniques conjuguées*, Bull. Sci. Math. vol. 56 (1932)), a set of $n$ conjugate harmonic functions $x_j$ of $n$ independent variables is a set such that (1) the $x_j$ are harmonic and (2) the function $-\sum(x_j+a_j)^{(2-n)/2}$ is harmonic or $-\infty$ for all values of the parameters $a_j$. It is now shown that there is a redundancy in the above characterization, for condition (1) is implied by condition (2). The result is extended by means of subharmonic functions to sets of $m$ conjugate harmonic functions of $n$ independent variables, $m \geqq n$. (Received December 31, 1942.)

### 110. R. H. Cameron and W. T. Martin: *An expression for the solution of a class of nonlinear integral equations.*

The authors give an expression for the solution of the integral equation $\phi(x)=f(x)+\int_0^x F[x, \xi, \phi(\xi)]d\xi$, where $F(x, \xi, y)$ is continuous in $0 \leqq x \leqq 1$, $0 \leqq \xi \leqq 1$, $-\infty < y < \infty$ and satisfies the uniform Lipschitz condition $|F(x, \xi, y_2)-F(x, \xi, y_1)| < M|y_2-y_1|$. For any continuous function $f(x)$ the solution $\phi(x)$ is given as the limit in the mean (in the $L_2$-sense) of the quotient of two Wiener integrals (averages) over the space of all continuous functions. The proof is carried through for more general nonlinear functional equations which include the above as a special case. (Received January 30, 1943.)

### 111. M. M. Day: *Uniform convexity.* III.

This paper fills out certain results obtained by the writer in two earlier papers (Bull. Amer. Math. Soc. vol. 47 (1941) pp. 313–317 and pp. 504–507). It also contains the following theorem: If a normed vector space is uniformly convex in the neighborhood of a single point on the unit sphere, then it is isomorphic to a uniformly convex space. (Received January 23, 1943.)

### 112. George Piranian: *On the convergence of certain partial sums of a Taylor series with gaps.*

Let $f(z)$ be defined by the series $\sum_{n=1}^{\infty}c_nz^{\lambda_n}$ where lim sup $|c_n|^{1/\lambda_n}=1$, and let $\theta_n=\lambda_{n+1}/\lambda_n-1$, $M(r)=\max_{|z|=r}|f(z)|$, and $S_n(z)=\sum_{p=1}^{n}c_pz^{\lambda_p}$. If lim sup $\{\log [M(1-\theta_{n_i}^2)/\theta_{n_i}]/\lambda_{n_i}\theta_{n_i}^2\} < \infty$, then lim $S_{n_i}(z)=f(z)$ at all regular points of $f(z)$ on the circle $|z|=1$. (Received December 12, 1942.)

### 113. R. M. Robinson: *Analytic functions in circular rings.*

The fundamental lemma on which this paper depends is the following: *If $f(z)$ is regular and single-valued in the ring $q \leqq |z| \leqq 1$, except for one simple pole on the negative real axis, and if $|f(z)| \leqq 1$ on both boundaries, then $|f(z)| < 1$ for $q < |z| < 1$, that is, on* *the radius opposite the pole.* Various applications are given, including the determination of the sharp bound in Hadamard's three circles theorem. That is, we suppose that $f(z)$ is regular and single-valued for $q \leqq |z| \leqq 1$, that $|f(z)| \leqq p$ for $|z| = q$ and $|f(z)| \leqq 1$ for $|z| = 1$, and find the largest possible value for $|f(z_0)|$, where $z_0$ is some point within the ring. A formula for the bound is given in terms of theta functions, and the problem is also discussed geometrically. In particular, if $q < p < 1$, then the maximum value of $|f(z_0)|$ is attained by a function $f(z)$ which is univalent in $q < |z| < 1$, and maps this ring on $|w| < 1$ excluding an arc of $|w| = p$. (Received January 23, 1943.)

### 114. Raphael Salem: *Sets of uniqueness and sets of multiplicity.*

An algebraic integer $\alpha$ having the property that all its conjugates have their moduli inferior to 1 will be called a "Pisot number" ($\alpha$ is necessarily real and greater than 1). The following theorems are proved: I. Let $0 < \xi < 1$. If the Fourier-Stieltjes transform $\prod_{k=0}^{\infty} \cos \pi u \xi^k$ does not tend to zero for $u \rightarrow \infty$, then $1/\xi$ is a Pisot number. II. Let $0 < \xi < 1/2$, and let $P$ be the symmetrical perfect set of Cantor type and of constant ratio of dissection $\xi$ constructed on $(0, 2\pi)$ (relative length of the black intervals is $1-2\xi$). Then $P$ is a set of uniqueness for trigonometrical series if (and only if) $1/\xi$ is a Pisot number. III. There exist Pisot numbers of the form $2+\epsilon$, $\epsilon$ being positive and arbitrarily small; hence, there exist sets of uniqueness which are of Hausdorff dimensionality as near to 1 as desired. (Received January 11, 1943.)

### 115. Gabor Szegö: *On the oscillation of differential transforms.* IV. *Jacobi polynomials.*

Let $\alpha \geqq 0$, $\beta \geqq 0$, $c \geqq 0$. In a recent paper (Trans. Amer. Math. Soc. vol. 52 (1942) pp. 463–497, cf. p. 489), E. Hille proved the following two theorems: (A) The differential operator $\vartheta - c = (1-x^2)D^2 + [\beta - \alpha - (\alpha+\beta+2)x]D - c$, $D = d/dx$, does not diminish the number of the sign changes of a function in $-1 < x < +1$; (B) If the number of the sign changes of $(\vartheta - c)^k f(x)$ remains less than or equal to $N$ for all $k$, $k = 1, 2, 3, \cdots$, then $f(x)$ is a polynomial of degree less than or equal to $N$. The purpose of the present note is the extension of Theorem A to $\alpha > -1$, $\beta > -1$ and of Theorem B to arbitrary real values of $\alpha$ and $\beta$, in the latter case with the modification that the possible degree of the polynomial $f(x)$ is less than or equal to $N + \gamma$, $\gamma = \gamma(\alpha, \beta, c)$. (Received January 20, 1943.)

## Applied Mathematics

### 116. Stefan Bergman: *A formula for the stream function in compressible fluid flow.*

Using the hodograph method and a general representation for the stream function of a flow of an incompressible fluid (see Bergman, *Hodograph method in the theory of compressible fluid*, Publication of Brown University, 1942) the author gives an explicit formula for the stream functions of flows of certain types. (Received January 27, 1943.)

### 117. Nathaniel Coburn: *Boundary value problems in plane plasticity.* Preliminary report.

The following problem is discussed in this paper: given an infinite plate of perfectly plastic material bounded by the $x$-axis; to determine the stresses within the plate when the stresses on the boundary are known. First, the equation of plasticity (yield