

vol. 5 (1939) pp. 784-788), the general canonical line as the line of intersection of the osculating planes of the projective geodesics in the directions \bar{D}_k results. (Received October 2, 1942.)

83. Y. C. Wong: *Some Einstein spaces with conformally separable fundamental tensors.*

When the fundamental tensor $*g_{\alpha\beta}$ of a Riemannian m -space is of the form $*g_{ij} = [\rho(x^\alpha)]^{-2}g_{ij}(x^k)$, $*g_{ip} = 0$, $*g_{pq} = [\sigma(x^\alpha)]^{-2}g_{pq}(x^r)$, $\alpha, \beta = 1, \dots, m$; $i, j, k = 1, \dots, n$; $p, q, r = n+1, \dots, m$, it is said to be conformally separable; $*g_{ij}$ and $*g_{pq}$, with x^r and x^k , respectively, as parameters, are called its component tensors. The author studies in this paper the conformally separable tensor which is the fundamental tensor of an Einstein m -space and each of whose component tensors either is of dimension less than three or is a family of fundamental tensors of Einstein spaces. It is found that the constructions of such a conformally separable tensor is invariably reduced to the construction of the fundamental tensor g_{ij} of an Einstein n -space or a Riemannian 2-space for which the equation $y_{,ij} = -I g_{ij}$ admits a non-constant solution for y , where the comma denotes covariant differentiation with respect to g_{ij} and I is an unspecified scalar. The author is content with this result, because the latter problem has already been considered in detail by H. W. Brinkmann in his study of Einstein spaces which are conformal to each other. (This paper will be published in the Trans. Amer. Math. Soc.) (Received October 2, 1942.)

NUMERICAL COMPUTATION

84. H. E. Salzer and Abraham Hillman: *Exact values of the first 120 factorials.*

Due to their fundamental importance, the exact values of the first 120 factorials were computed and checked. $120!$ contains 199 digits. $100!$ agreed with Uhler's value (Proc. Nat. Acad. Sci. U.S.A. vol. 28 (1942) p. 61). When these values were compared with Potin's table of the first 50 factorials (*Formules et tables numériques*, p. 836) errors were found in Potin's values for $18!$, $38!$, $45!$, and $50!$. (Received November 11, 1942.)

STATISTICS AND PROBABILITY

85. J. H. Curtiss: *A note on the theory of moment generating functions.*

The moment generating function (m.g.f.) of a variate X is defined as the mean value of $\exp(\alpha X)$, the characteristic function (c.f.) as the mean value of $\exp(itX)$, where α and t are real. The purpose of this note is to place on record careful statements and proofs of the appropriate analogues for the m.g.f. of the well known uniqueness and limit theorems for the c.f. For example, Levy's continuity theorem assumes the following form: Let $F_n(x)$ and $G_n(\alpha)$ be, respectively, the d.f. and m.g.f. of a variate X_n . If $G_n(\alpha)$ exists for $|\alpha| < \alpha_1$ and for all $n \geq n_0$, and if there exists a function $G(\alpha)$ defined for $|\alpha| \leq \alpha_2 < \alpha_1$, $\alpha_2 > 0$, such that $\lim_{n \rightarrow \infty} G_n(\alpha) = G(\alpha)$ uniformly, $|\alpha| \leq \alpha_2$, then there exists a variate X with d.f. $F(x)$ such that $\lim_{n \rightarrow \infty} F_n(x) = F(x)$ uniformly in each finite interval of continuity of $F(x)$. The m.g.f. of X exists for $|\alpha| \leq \alpha_2$ and is equal to $G(\alpha)$ in that interval. (Received October 9, 1942.)

86. W. K. Feller: *On a probability limit theorem of H. Cramér.*

Let $\{X_k\}$ be mutually independent random variables whose first moments vanish and whose second moments are σ_k^2 ; let $s_n^2 = \sigma_1^2 + \cdots + \sigma_n^2$. In various applications one is concerned with the distribution function $Pr\{X_1 + \cdots + X_n > xs_n\}$, where $x \rightarrow \infty$ as $n \rightarrow \infty$. The simplest binomial case has been studied by A. Khintchine, P. Lévy, Smirnov and others. H. Cramér found a complete description of the asymptotic behavior of the above sums in the case where all X_k have the same distribution function. This theorem is generalized to the case of unequal components. The theorem is to serve as a base for a solution of the "problem of the iterated logarithm" in the general case. (Received November 21, 1942.)

87. W. K. Feller: *On the general form of the so-called law of the iterated logarithm.*

Let $\{X_k\}$ be a sequence of mutually independent random variables whose first moments vanish and whose second moments are σ_k^2 ; let $s_n^2 = \sigma_1^2 + \cdots + \sigma_n^2$. A sequence of numbers $\phi_k \uparrow \infty$ is said to be of upper (lower) class if the probability that $X_1 + \cdots + X_n > s_n \phi_n$ for infinitely many k is one (zero); any sequence $\{\phi_k\}$ is either of upper or of lower class. A n.a.s. condition is found for a sequence $\{\phi_k\}$ to belong to the upper class. It generalizes the condition found by Kolmogoroff and Erdős in the special case where X_k assumes the values ± 1 only, each with probability $1/2$; however, it is different in form. The new theorem also contains a result of Marcinkiewicz and Zygmund on the necessity of the condition imposed by Kolmogoroff on the X_k in his proof of the law of the iterated logarithm. (Received November 21, 1942.)

88. P. G. Hoel: *On indices of dispersion.*

The sampling distribution of the index of dispersion for binomial and Poisson distributions is investigated by means of semi-invariants. Approximations to terms of order N^{-3} are obtained for the descriptive moments of the distribution, by means of which the accuracy of the chi-square approximation can be determined. (Received October 30, 1942.)

TOPOLOGY

89. R. F. Arens: *Homeomorphism groups of a space.* Preliminary report.

Let A be a locally bicomact, locally connected Hausdorff space, and let G be a group of homeomorphisms of A . Then there is a certain topology N making G into a topological group (Pontrjagin, *Topological groups*, Princeton, 1939). This topology N is the weakest admissible topology that can be introduced into G , in this sense: Sets in G open by N are open by any other admissible topology M . A topology M for a group of homeomorphisms, G , of a space A is called admissible if by using that topology for G the two functions $g(a)$ and $g^{-1}(a)$, where $g \in G$ and $a \in A$, become continuous functions of both arguments g and a simultaneously. The topology N is determined by the following system of neighborhoods of the identity in G : Select in A an open set W whose closure is bicomact, and another open set whose closure K is contained in W . Then the set U of all $g \in G$ which transform K into W is defined to be a neighborhood of the identity. The set of all such U together with all their finite