$L(S)=L(S_1)+L(S_2)$ (Rado and Reichelderfer, *On a stretching process for surfaces*, Amer. J. Math. vol. 61 (1939)). (Received November 28, 1942.)

57. Antoni Zygmund: *A property of the zeros of Legendre polynomials.*

Suppose that $n<m$ are positive integers and that a polynomial $\phi(x)$ of degree n does not exceed M in absolute value at the zeros of the Legendre polynomial $P_m(x)$. Then $|\phi(x)|\leqq A(\delta)M$ for $-1\leqq x\leqq +1$, where $A(\delta)$ depends only on the number δ defined by the equations $m/n=1+\delta$. Similar results hold for the integrals $\int_{-1}^{+1}|\phi(x)|^r dx$ with $r\geqq 1$. (Received November 23, 1942.)

Applied Mathematics

58. Stefan Bergman: *A formula for the stream function of compressible fluid flow.*

Let $\mathfrak{q}=ve^{i\theta}$ denote the velocity vector. A flow, $\mathcal{J}$ is said to be of the type D_n, if the boundary of the domain, B, in which $\mathcal{J}$ is defined consists of $2n$ segments S_K such that along each S_{2K}, $K=1, 2, \cdots, n$, $\theta=\theta_K$ is constant and along each S_{2K-1}, v is constant. (S_{2K} are segments of straight lines, S_{2K-1} are so-called "free boundaries.") The image of B in the logarithmic plane (see *Notes on hodograph method in the theory of compressible fluid*, publication of Brown University, p. 6) is a polygonal domain. In the case of an incompressible fluid the stream function of $\mathcal{J}$ can be represented as a closed expression with n parameters. The author considers subsonic flows, $\mathcal{C}$, of compressible fluid. Using certain linear operators (see above mentioned *Notes*, §§6 and 10, and Trans. Amer. Math. Soc. vol. 53 (1943) pp. 130–155) he derives a similar explicit formula with n parameters for the flows $\mathcal{C}$ of "nearly type D_n," that is to say, for flows whose boundaries consist of $2n$ segments along which θ or v assume nearly constant values. The angles θ_K may be prescribed. (Received November 21, 1942.)

59. R. M. Foster: *On the average resistance of an electrical network.*

In an electrical network composed of two-terminal resistance elements, let $J\cdot$ designate the total resistance measured across the terminals of an element, the internal resistance of this element being r_i, and let S_i be the driving-point resistance measured in the branch containing this element r_i. It is shown in this paper that, for any network configuration whatsoever, $\sum J_i/r_i=R$ and $\sum r_iS_i=N$ (the summation being extended over all the elements of the network), with $R=V-P$ and $N=E-V+P$, where E is the number of elements, V the number of vertices, and P the number of separate, unconnected parts of the configuration. The average values of the ratios J_i/r_i and r_i/S_i are thus R/E and N/E, respectively. If all the elements of the network have the same internal resistance r, and if there is complete symmetry among the elements so that the resistance measured across any one element is necessarily equal to that across any other element, then $J_i=rR/E$ and $S_i=rE/N$. These results are extended to generalized impedances, and to infinite networks. (Received November 23, 1942.)

60. A. H. Fox: *Integral representation of the flow of a compressible fluid around a cylinder.*

The steady irrotational two-dimensional flow of a compressible fluid may be approximated by the flow of a hypothetical incompressible fluid in which the pressure is represented by a polynomial in the reciprocal of the density. The stream function of the flow may be expanded in a series in terms of a parameter q that appears in the polynomial. For $q=0$ this reduces to the von Kármán-Tsien method of attacking similar problems. The differential equation of the flow in the hodograph plane may be reduced to the form $\psi^*_{z\bar{z}}+F\psi^*=0$, for which the integral representation ψ^* is the real part of $\int_{-1}^{1}E^*(z, \bar{z}, t)(1-t^2)^{-1/2}f[1/2z(1-t^2)]dt$ has been established by Bergman. The function f is a function of one complex variable determined by the pattern of the flow in the hypothetical fluid. In the present paper, the method is applied to the flow of compressible fluid around a circular obstacle. (Received November 24, 1942.)

61. K. O. Friedrichs: *On nonlinear vibrations of third order.*

Problems of self-excited oscillations frequently lead to differential equations of higher than second order. While for the order two, Bendixson's theorem furnishes a general criterion for the existence of periodic solutions, no similar general criterion is available in cases of higher order. The present paper analyzes such a problem of order three (occurring in the theory of electrical oscillations in circuits involving vacuum tubes). First, the existence of periodic solutions is obtained from the fixed-point theorem by a special construction. In addition, periodic solutions are investigated in the neighborhood of degenerate cases of order two by a perturbation method, which yields asymptotic expansions. (Received November 23, 1942.)

62. Hilda Geiringer: *New convergence cases for iteration methods applied to linear equations.*

To obtain the solution $x^{\rightarrow}$ of the system $\sum_{k=1}^{n}a_{ik}x_k+r_i=0$ $(i=1, \cdots, n; |A|\neq 0)$, by iteration through "successive displacements" new values $x_i^{(\nu+1)}$ are successively computed from the ith equation using $x_1^{(\nu+1)}, \cdots, x_{i-1}^{(\nu+1)}, x_{i+1}^{(\nu)}, \cdots, x_n^{(\nu)}$, $(\nu=0, 1, 2, \cdots; x^{(0)\rightarrow}$ arbitrary). (See Bull. Amer. Math. Soc. abstract 48-5-202.) The only sufficient condition for the convergence of $x^{(\nu)\rightarrow}\rightarrow x^{\rightarrow}$ known so far is that the matrix (a_{ik}) is positive definite. In this case the procedure reduces to Southwell's relaxation-method. Now the necessary and sufficient condition for convergence is that the roots ρ_i of a certain equation $F_{n-1}(\rho)=0$ of degree $n-1$ are all less than one in absolute value, a condition assuming no symmetry. If, however, the matrix is symmetric and $a_{ii}>0$ then positive-definiteness is necessary for convergence from an arbitrary starting point $x^{(0)\rightarrow}$. In addition it is proved that all sufficient conditions known for the "ordinary" iteration, that is, for the iterated linear homogeneous transformation of the "error-vector" $z_i^{(\nu)}=x_i^{(\nu)}-x_i$, assure convergence of the successive displacements, but by no means vice versa; and a new sufficient condition for both methods is established. In all these cases any order is admissible for the successive displacements; also the convergence may be improved by "group-iteration." (Received November 23, 1942.)

63. A. E. Heins: *Some remarks on the solution of dual integral equations.* I.

The solutions of dual integral equations whose kernels can be expressed in a series of Bessel functions are considered. Explicit solutions for these equations are given and their properties are discussed. (Received November 23, 1942.)

64. Max Herzberger: *Direct methods in geometrical optics.*

The author presents a new approach to the problem of geometrical optics deviating from the methods of W. I. Hamilton. Hamilton has shown the existence of a characteristic function, the knowledge of which gives all the information necessary about an optical system. However, the construction of Hamilton's function for any given optical system seems to lead to insurmountable mathematical difficulties. The author proposes instead to calculate the coordinates of the image rays directly as functions of the coordinates of the object rays. Instead of getting a single function like Hamilton, he finds four functions connected by three differential equations and one finite equation. He has succeeded in obtaining the explicit form of these functions for any given optical system and developing an image error theory which is more adapted to the practical problems of lens design. (Received October 3, 1942.)

65. Fritz John: *Instability of certain nonlinear vibrations.*

The differential equation $(E): x''(t)+f(x)=-F\sin\lambda t$, where $f(0)=0$, g.l.b. $f'(x)>0$, λ and F positive constants, represents the motion of a spring with nonlinear restoring force and without damping under a periodic disturbing force. It is shown here: For every $M>0$ there exists and is uniquely determined a value $\lambda>0$ and a solution $x(t)$ of (E) of period $2\pi/\lambda$, for which $x=M$, $x'=0$ for $t=\pi/2\lambda$ and $x'>0$ for $|t|<\pi/2\lambda$ ($x(t)$ then is a periodic solution of phase difference π/λ). The value λ determined by M may define the "resonance function" $\lambda=\lambda(M)$ (this is only one branch of the complete resonance curve). For $d\lambda/dM>0$ the corresponding periodic solutions $x(t)$ are shown to be unstable in the following sense: there is an $\epsilon>0$, such that for every $\delta>0$ there exists a solution $x_1(t)$ of (E), for which $|x_1-x|<\delta$, $|x_1'-x'|<\delta$ for some t, and $|x_1-x|>\epsilon$ for other t. One always has $d\lambda/dM\leqq 0$ for "soft" springs, that is, if $f'(x)\leqq f(x)/x$. (Received November 23, 1942.)

66. Edward Kasner: *Trajectories in a resisting medium.*

Consider the motion of a particle moving in a general positional field and influenced by a resisting medium R. The five fundamental properties of a pure positional field (*Differential-geometric aspects of dynamics*, Amer. Math. Soc. Colloquium Publications vol. 3, 1913, chap. 1) are of course in general changed. However if $R=Av^2+B$, property I remains valid. If properties I and II are valid then the resistance is found to vary as the square of the speed. If I, II, III hold then R vanishes. The discussion is carried out for both two- and three-dimensional fields. Fields of simple and complicated character are studied where either all, or the maximum infinitude, of trajectories are circles (a particular example is the Maxwell central force). (Received November 18, 1942.)

67. Arthur Korn: *On vibrational vortices.*

The vortex equations in fluids are nonlinear differential equations, but in some special cases they may be replaced by linear equations. A special case is the problem of vibrational velocities (vectors): $u=u_0+u_1\cos\nu t+u_2\sin\nu t$, where ν is an exceedingly great frequency and the first derivatives of u_0, u_1, u_2 with respect to the time t and the first derivatives with respect to the coordinates x, y, z of u_1, u_2 can be neglected in comparison with νu_0, νu_1, νu_2. Then one remarks that on the left-hand side of the vortex equations the main terms are $(-\text{curl } u_1\sin\nu t+\text{curl } u_2\cos\nu t)\nu/\rho$, ρ being the density of the fluid. In order to obtain on the right-hand side such great terms, the first derivatives of u_0 with respect to x, y, z must be exceedingly great. Excluding exceedingly great values of $\rho u_0^2/2$ such cases cannot be realized in incompressible fluids, but they are perfectly possible in compressible fluids, even if the conditions div $u_1=0$ and div $u_2=0$ are retained. u_0 must have wave character with an exceedingly small wave length λ. Such cases can be handled by means of linear differential equations. (Received November 21, 1942.)

68. Brockway McMillan: *Networks of mechanisms*. Preliminary report.

A mechanism M maps a class I of input histories $i(t)$ upon a class O of output histories $o(t)$, $-\infty < t < \infty$. It is single-valued and has the property that whenever $i_1(t) \equiv i_2(t)$ for $t \leqq t_o$, then $o_1(t) \equiv o_2(t)$ for $t \leqq t_o$. If $o_1(t) \equiv o_2(t)$ for $t \leqq t_o + \lambda$, uniformly in i_1, i_2, and t_o, then M has the latency λ. Suppose that I is closed under an operation of addition, that a null function $i_o(t) \equiv \phi$ is in I, that every $i(t)$ is identically ϕ near $t = -\infty$, and that $O \subseteq I$. Let (M_k) be a collection of mechanisms from I to O such that (a) each has latency at least $\lambda > 0$, and (b) each maps the null function on itself. The inputs and outputs of the M_k are interconnected to form a network N. Arbitrary inputs $i(t) \in I$ at each junction make the components of a vector input to N. The vector output of N has for its components the outputs of the various M_k. Theorem: I and O can be extended so that N is a mechanism between its vector inputs and outputs, with latency λ. A motivation is the possibility of application to nerve fiber networks. (Received November 4, 1942.)

69. W. H. Roever: *A new formula for the deviation in range of a projectile due to the earth's rotation.*

On page 68 in his monograph entitled, *The weight field of force of the earth*, published in the Washington University Studies, September, 1940, the author derives for the range of a projectile, a formula [second part of (129)] which by a simple trigonometric transformation can be put in the new form $\bar{x} = (v_0^2/g_1) \sin 2\beta + \Delta\bar{x}$ where $\Delta\bar{x} = -(4v_0^3/3g_1^2)\, \omega \cos \phi_1 \sin 3\beta \sin \alpha$, in which ω is the angular velocity of the earth's rotation, g_1 is the acceleration, due to weight, at the position of the gun, ϕ_2 is the astronomical latitude of the position of the gun, α is the azimuth (measured from the south through the west) of the direction of fire, β is the angle of elevation of the gun, v_0 is the muzzle velocity of the projectile, he points out particularly that for fixed values of α and ϕ_1, $\Delta\bar{x}$ changes sign when $\beta = 60°$. (Received November 23, 1942.)

Geometry

70. John DeCicco: *Conformal geometry of second order differential equations.*

Kasner in his fundamental paper, *Conformal geometry*, Prcoeedings of the International Congress of Mathematicians, 1912, initiated the conformal study of sets of analytic curves. In previous work, Kasner (with the author) studied the conformal geometry of velocity systems of curves $y'' = (1+y'^2)[\phi(x, y) + y'\psi(x, y)]$. This class of velocity systems characterizes the conformal group. Any velocity system possesses six absolute conformal differential covariants of second order. In this paper, it is shown that a system of ∞^2 curves, not of the velocity type, possesses three absolute conformal differential covariants of third order. Moreover any other conformal covariant is a function of these and their partial derivatives. Geometric interpretations of these covariants are also obtained. (Received November 21, 1942.)