ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA AND THEORY OF NUMBERS

299. Leonard Carlitz: *Representation of a polynomial in certain forms.*

Explicit formulas are found for the representation of an irreducible polynomial P in $GF(p^n)$ in certain factorable forms of degree k in k indeterminates. For example, when $k=2$, $p\neq 2$, the form may be taken as $U^2-\alpha V^2$, where α is not a square in $GF(p^n)$. Here U is determined by the formula $2A(x)\equiv(-1)^h[1][3]\cdots[2h-1]$ (mod $P(x)$), where P is of degree $2h$, and $[i]=x^{p^{ni}}-x$ (Duke Mathematical Journal, vol. 9 (1942), pp. 234–243, p. 242). Similar results are obtained for the general case. (Received August 3, 1942.)

300. Leonard Carlitz: *Some formulas for the composition of numerical functions.*

This note is concerned with the sum $h(M)=\sum f(U)g(V)$ extended over polynomials of fixed degree k in $GF(p^n)$ such that $(\alpha+\beta)M=\alpha U+\beta V$, α, β in $GF(p^n)$; it is assumed that $f(U)$ and $g(V)$ are of the form $\sum e(D)$, where D runs through the divisors of U of degree at most $k/2$. It is found that $h(M)$ is of the same general form. In various special cases $h(M)$ can be expressed in quite simple form. In particular the formulas of the present paper include formulas occurring in the problem of representing a polynomial as the sum of an even number of squares. (Received August 3, 1942.)

301. R. P. Dilworth: *On the decomposition theory of modular lattices.*

The Kurosch-Ore decomposition theorem asserts that if an element a of a modular lattice has two reduced decompositions into irreducibles $a=p_1\cap\cdots\cap p_m=q_1\cap\cdots\cap q_n$, then each p_i may be replaced by a suitably chosen q_j. However, this leaves unanswered questions like the following: Does each q_j replace some p_i? Can different p's be replaced by different q's? The following precise result is proved: If $a=p_1\cap\cdots\cap p_m=q_1\cap\cdots\cap q_n$, then the q's can be renumbered in such a way that each p_i may be replaced by q_i. This theorem requires a much deeper analysis of the arithmetical structure of a modular lattice. For this purpose extensive use is made of the concept of an *over-divisor* which is defined as follows: a is said to *over-divide* b, or to be an *over-divisor* of b if $x\cap a=b$ implies $x=b$. (Received August 4, 1942.)

302. C. J. Everett: *Sequence completion of lattice moduls.*

A lattice modul L (G. Birkhoff, *Lattice ordered groups*, Annals of Mathematics, (2), vol. 43) is linear if and only if $a>0$, $b>0$ implies $a\wedge b>0$; hence the neighborhood topology $N_e(a)=(x;\ |x-a|<e,\ e>0)$ defines a topological group satisfying the crucial Hausdorff space intersection axiom if and only if L is linear. o-convergence and o-regular sequences are defined, and necessary and sufficient conditions given for every o-regular sequence to o-converge. Such an L is called o-complete. The first fundamental extension L' of L by classes of regular sequences imbeds L with preservation of $+$, unrestricted $\vee$, $\wedge$, and o-convergence. If L admits diagonalization (G. Birkhoff, *Lattice Theory*, American Mathematical Society Colloquium Publications, vol. 25, 1940, p. 28), then L' is o-complete. The MacNeille completion $\mathfrak{C}$ of the lattice L by cuts contains a maximal lattice submodul $\mathfrak{M}\supseteq L$ of all elements of $\mathfrak{C}$ having inverses. $\mathfrak{M}$ is o-complete and imbeds L with the same preservations as for L'. L' is isomorphic to the set L_1 of the elements of $\mathfrak{M}$ which are o-limits of L-sequences relative to L similarly L'', L''', and so on. A sequence is o-regular in L if and only if it o-converges in $\mathfrak{M}$ relative to L. If L is linear, o-convergence is equivalent to sequence convergence relative to $N_e(a)$ topology; L_1 is o-complete, indeed $L_1=\mathfrak{M}$ whenever there exist nonzero null sequences in L. (Received October 1, 1942.)

303. Irving Kaplansky: *Systems of congruences in a valuation ring.*

Infinite systems of linear congruences, introduced by Krull in his general definition of completeness of fields with valuations, are here used to obtain a characterization of maximality. As an application, a classical result is generalized; under the hypothesis that the field K is maximal in a valuation, it is shown that the degree of any extension of K is equal to the product of the ramification order and residue class degree. (Received September 29, 1942.)

304. Irving Kaplansky: *The direct product of rings.*

Whitney's tensor product $G\circ H$ of abelian groups (Duke Mathematical Journal, vol. 4 (1938), pp. 495–528), reduced with respect to a ring of operators R, provides a natural definition of the direct product of rings. If G and H are imbedded in a larger ring, the problem arises of deducing the usual criterion that the product GH be direct; this can be done if certain restrictions are made, as indicated by Dorroh in the case where R is the ring of integers (Annals of Mathematics, (2), vol. 36 (1935), pp. 882–885). This general notion of direct product is used to obtain a variety of results, some generalizations of known theorems, others new. (Received September 29, 1942.)

305. A. J. Kempner: *Periodic decimals to any base, and quadratic residues.*

Despite the large literature on the periods of decimal fractions there does not seem to exist any unified theory of this very special field. Restriction to the base 10 or to any individual base q is one reason for this deficiency. In order to gain a broader basis it is necessary to consider the representation of *all* proper fractions for *all* bases. In this enlarged formulation of the problem there is room for a satisfying theory. To bring out the structure of the system it is of advantage to place emphasis on the system of remainders $a_1, \cdots, a_\mu$ rather than on the system of digits $\alpha_1, \cdots, \alpha_\mu$ in the algorithm for the expansion of a/b to base q ($a\cdot q=\alpha_1\cdot b+a_1$, $a_1\cdot q=\alpha_2\cdot a_1+a_2, \cdots, a_{\lambda-1}\cdot q=\alpha_\lambda\cdot a_{\lambda-1}+a_\lambda, \cdots$). The a's possess a period of the same length μ as do the α's, but have the advantage of being all distinct in a period, thus leading to a more incisive classification than is possible with the digits. (Received August 4, 1942.)

306. Gordon Pall: *The distribution of integers represented by binary quadratic forms.*

The formula due to R. D. James (American Journal of Mathematics, vol. 60 (1938), pp. 737–744) for the number of integers m prime to d represented by binary quadratic forms of discriminant d, is here freed of the restriction that m be prime to d. (Received August 3, 1942.)

307. Gordon Pall: *The weight of an n-ary genus of quadratic forms.*

The formula for the weight of a genus of integral positive quadratic forms in n variables is obtained in an explicit, useful form. The calculation of the factor for $p=2$ is greatly facilitated by the use of a much simplified system of invariants. There are numerous applications. (Received August 3, 1942.)

308. J. F. Ritt: *Bezout's theorem and algebraic differential equations.*

The intersection of the general solutions of two differential polynomials in two unknowns is examined with respect to the numbers of arbitrary constants on which its various irreducible components can depend. (Received August 6, 1942.)

309. Ernst Snapper: *The resultant of a linear set.*

The m-dimensional vector space $\overline{V}_m$ consists of vectors having, as components, m polynomials of the ring $P[y_1 \cdots y_n]$ where P is a field. The linear subsets of $\overline{V}_m$ are generated by the columns of $m \times s$ matrices with elements in $P[y_1 \cdots y_n]$. The ideal theory of $P[y_1 \cdots y_n]$, given by Hentzelt and Noether (Mathematische Annalen vol. 88 (1922), pp. 53–79), holds for these linear sets. By a linear, invertable transformation of the variables $y_1, \cdots, y_n$, which involves adjoining new variables $\gamma_{i,j}$ to P, the linear subsets of $\overline{V}_m$ become "transformed" linear sets of the vector space V_m over $P(\gamma)[x_1 \cdots x_n]$. Every transformed linear set L of V_m has a resultant $\rho \in P(\gamma)[x_1 \cdots x_n]$, which vanishes for, and only for, the zeros of the ideal $L/\overline{L}$. (See Snapper, Transactions of this Society, vol. 52 (1942), pp. 258–259 for the definitions of $\overline{L}$ and $L/\overline{L}$.) If $L_1 \subseteq L_2$, then $L_1 = L_2$, if and only if they have equal ranks and resultants. This gives a criterion for the existence of a polynomial solution of simultaneous linear equations with polynomials as coefficients. For $n=1$, the resultant becomes the highest dimensional determinantal factor of L. (Received September 29, 1942.)

ANALYSIS

310. M. A. Basoco: *On the Fourier developments of a certain class of theta quotients.*

This paper is concerned with the functions $\phi_\alpha^k(z) \equiv \{\vartheta_\alpha'(z, q)/\vartheta(_\alpha z, q)\}^k$ $(\alpha = 0, 1, 2, 3)$ where $\vartheta_\alpha(z, q)$ is a Jacobi theta function and k is a positive integer. The Fourier expansions of these functions are investigated and their arithmetical form is obtained for the cases $k=1, 2, 3$. Using these results and certain simple identities, there is obtained using the method of paraphrase (E. T. Bell, Transactions of this Society, vol. 22 (1921), pp. 1–30 and 198–219; *Algebraic Arithmetic*, American Mathematical Society Colloquium Publications, vol. 7, 1927, chap. 3), a series of theorems on numer-