cycles. It also gives a new "economic" interpretation of history and presents some interesting analogies between mathematics and economics.

This book should prove very useful to the statistician as a handbook. He will find a great deal of stimulation and valuable material and procedures even if he is not professionally concerned with economic problems. The pure mathematician can gain a survey of the field. He may be interested especially in some of the unsolved problems of serial correlation and periodogram analysis which evidently require very powerful tools of analysis. One has to be extremely grateful to the author for having accumulated so much material and presented it in such a readable and interesting form.

GERHARD TINTNER

A Survey of Modern Algebra. By Garrett Birkhoff and Saunders MacLane. New York, Macmillan 1941. 11+450 pp. $3.75.

In this book Professors Birkhoff and MacLane have made an important contribution to the pedagogy of algebra. Their emphasis is on the methods and spirit of modern algebra rather than on the subject matter for itself. The word "survey" in the title is quite accurate; for, although many topics are treated, none of them is really completely developed. The most important parts of each theory are included and that is all that can be asked of an introductory textbook.

Because of the authors' emphasis on "method" rather than "fact" the book will not be of much use as a reference work. But there is no dearth of good reference works in algebra, and in the reviewer's opinion the present textbook will prove more useful than another encyclopedic treatise would have been.

A discussion of the topics included will help to indicate the authors' purposes.

The first three chapters of the book are ostensibly devoted to the development of number systems: starting with postulates for the integers, then defining rational numbers in terms of the integers, and next giving an outline of the Dedekind construction of real numbers. Actually much more is happening. Such fundamental concepts as congruence, residue class, isomorphism, and ordered and well ordered sets are introduced and applied in a natural manner to the theory. Also the generalizations from integers to integral domain, and from rational number to field are made at suitable stages of the development. This procedure of starting with properties of a familiar mathematical system and generalizing to an abstract system is typical of

the spirit of the book and is used in the development of almost every topic treated.

The traditional theory of equations, which is all too frequently the undergraduate's only contact with algebra, places much emphasis on the solution of polynomial (and sometimes also non-algebraic) equations by approximation methods. This involves considerable attention to the analysis of polynomial functions, including such topics as Descartes' rules of signs, and Sturm's and Budan's theorems for isolation of roots. Such theorems may indeed be nice applications of the calculus but they certainly do not give the student any new general concepts either in algebra or analysis. A result of this procedure is that most beginning graduate students have not even the ghost of an idea as to what algebra is really about, even those parts of algebra that have proved to be of almost universal mathematical interest. (Contrast this with the situation in analysis where the undergraduate is getting acquainted with such basic ideas as continuity, limit, and infinite processes.)

In contrast to the traditional approach, in the authors' treatment (Chapters IV and V) the parts of the theory of polynomials and theory of equations that belong to analysis are omitted. (One major exception to this general policy is the inclusion of several sections on the fundamental theorem of algebra, but these can be skipped without loss of continuity.) Because of these omissions, an undergraduate course in algebra based on this book has time to include numerous more generally useful algebraic topics and methods; that is, the student can be presented with ideas that lead somewhere, rather than being sent up a blind alley.

Chapter VI deals with group theory. Only the most elementary (and therefore the most frequently needed) parts of the theory of abstract groups are included. The importance of groups of transformations and of permutation groups is emphasized throughout. As usual the topics treated are pegs upon which are hung new general ideas: this time the concepts of automorphism, homomorphism, and a general discussion of abstract equivalence and congruence relations.

The authors express the belief that "for many students, the value of algebra lies in its applications to other fields: higher analysis, geometry, physics, and philosophy." This belief is reflected in many places throughout the book; but one of the most striking examples is found in their treatment of matrices and determinants (Chapters VII–X), which is so arranged as to be an excellent foundation for a later study of Hilbert and Banach spaces. Before matrices are men-

tioned at all the concepts of vector space, linear dependence, dimension (including infinite-dimensional spaces), inner product, euclidean vector space, normal orthogonal base are considered. Then matrices are introduced as arrays which describe linear transformations of a vector space (relative to some basis); and various properties of matrices are defined relative to the central concept of vector space. Linear groups are introduced as automorphisms of vector spaces. The full linear, the affine, the orthogonal, and the euclidean groups are studied. The problem of classifying quadratic forms relative to these groups is considered.

To clarify the two possible interpretations of change of basis the authors introduce the terms *alibi* (the point moves elsewhere) and *alias* (the point is renamed). This distinction pays rich dividends in the later discussions of various kinds of equivalence and canonical forms. While treating groups of transformations it is natural to discuss invariants under these groups. At this stage the authors insert a discussion of the place of canonical forms in invariant theory. In the reviewer's opinion these little general discussions that appear from time to time are one of the most valuable contributions of the book.

The last of these four chapters opens with a discussion on ranks of matrices and a treatment of the elementary properties of determinants. The authors' interest in applications of algebra to other fields is evidenced here by an interpretation of determinants as volumes. The chapter closes with a survey of the theory of similarity of matrices. Their development does not cover the case of a matrix whose minimum equation has repeated roots, but what they do give illustrates the general ideas and includes many classes of matrices important in other fields.

The recent wave of interest in lattice theory and partially ordered sets is reflected in Chapter XI. The first part of this chapter is a brief treatment of Boolean algebras; then lattices are introduced as a generalization. Chapter XII is devoted to transfinite arithmetic.

In Chapters XIII and XIV enough material is included to give the student the flavor of the theory of rings and fields, but for the real substance of these topics he will have to look elsewhere. The treatment of fields goes far enough to serve as a basis for the final chapter which gives Artin's development of the Galois theory of equations. Solvable groups are studied, and the insolvability by radicals of the general quintic equation is proved.

A very important feature of the book is the inclusion of many exercises. Most of these are illustrative in nature, but a goodly number involve useful extensions of the theory as developed in the text.

The authors' preface includes a summary of the purposes and contents of each chapter, and also indicates several levels of courses in which the book is designed to be used as a text. The most elementary of these is a one year undergraduate course based on only high school algebra. Provision is made for a semester course covering the tools used in physical applications. Finally, a semester course on abstract algebra can be carved out by selecting suitable chapters. The reviewer recommends the use of the book as a text in any of these ways.

The format is pleasing and the text seems to be remarkably free from typographical errors and minor slips.

R. M. THRALL