The proof consists in an asymptotic calculation of $U(x, \rho)$ based on the theory of asymptotic solution of ordinary linear differential equations involving a parameter, as developed by G. D. Birkhoff, Noaillon, Tamarkin, Trjitzinsky and others. (Received January 8, 1942.)

Applied Mathematics

122. Stefan Bergman: *Two-dimensional flow around two profiles.*

The study of the influence of tail surfaces on lift and pressure distribution of a wing can be reduced to the investigation of problems in the conformal mapping of doubly connected domains. The use of orthogonal functions enables one to give simple formulas for the lift and the moment in the case of a uniform flow around one and around two profiles. In the first case the lift, L, is found to be the expression $L=4(\pi)^{1/2}\rho V^2 (\sum |\phi_\nu(0)|^2)^{1/2} \sin [\alpha+\pi-\arg (\sum{}^*\psi_\nu(b)\phi_\nu(0))]$, where ρ is the density, $Ve^{-i\alpha}$ the velocity at infinity, $\psi_\nu(z)=\int_0^z\phi_\nu(z)dz$, and $\{\phi_\nu(z)\}$ a complete system of orthonormal polynomials of a domain B. (B is the domain obtained from the exterior of the profile by the transformation $z=1/\zeta$, b is the coordinate of the cusp, and the summation $\sum{}^*$ is understood in a certain special sense.) Analogous formulas exist for the moment and similar ones in the case of a flow around two profiles. (See also Notes of Lectures on Conformal Mapping, publication of Brown University, chap. XI, §§5–7) (Received January 29, 1942.)

123. Henry Wallman: *On the reduction in harmonic distortion due to high frequency pre-emphasis.* Preliminary report.

It is now common practice in high-fidelity sound broadcasting, in either FM or AM, to employ high frequency pre-emphasis, the object being an increase in signal-to-noise ratio. An additional effect, namely a reduction in harmonic distortion, has been noted experimentally. An analysis of this reduction in distortion is made in this paper, and quantitative evaluations are given for single-tone harmonic distortion of all orders. (Received December 30, 1941.)

124. Alexander Weinstein: *Spherical pendulum and complex integration.*

The following theorem, due to Puiseux (Journal de Mathématiques, 1842) is proved by a simple application of the theory of residues: The increment of the azimuth of a spherical pendulum corresponding to its passage from the lowest level z_1 to the highest level z_2 is greater than $\pi/2$. The boundary of the domain in the complex z-plane to which Cauchy's theorem is applied consists of a cut connecting z_1 with z_2 and of a vertical straight line to the right of z_2. (Received January 26, 1942.)

Geometry

125. P. O. Bell: *The parametric osculating quadrics of a family of curves on a surface.*

In this paper the author investigates the properties of the parametric osculating quadrics of a family of curves on a surface. These quadrics were introduced by Dan Sun (Tôhoku Mathematical Journal, vol. 32 (1930), pp. 81–85). His definition is essentially the following: At three neighboring points P, P_1, P_2 on an asymptotic curve C_u of a surface S construct the tangents to the curves of a one-parameter family on S.

The three tangents determine a quadric whose limit, as P_1, P_2 independently approach P along C_u, is the parametric osculating quadric $Q^{(u)}$ of the one-parameter family of curves of S at P. A quadric $Q^{(v)}$ is similarly defined with respect to the other asymptotic curve C_v. The union curves of an arbitrary congruence Γ', the curves of Darboux and the curves of Segre are all characterized geometrically in association with the quadrics $Q^{(u)}$ and $Q^{(v)}$. This paper will appear in the American Journal of Mathematics. (Received January 29, 1942.)

126. Nathaniel Coburn: *Conformal geometry of unitary space.*

The principal purpose of this paper is the determination of those affinors of a unitary K_n which: (I) transform as affinors under the analytic group of K_n; (II) are invariant under conformal transformations of the fundamental tensor of K_n. An equi-conformal fundamental tensor and connection are introduced, both of which are invariant under (II). It is shown that if the group of K_n is equi-analytic (determinants of (I) are constant), then the components of the equi-conformal connection transform as do the components of the ordinary connection of K_n. By use of the equi-conformal connection, a conformal connection is determined which transforms as does the ordinary connection of K_n under (I) and which is invariant under (II). Necessary and sufficient conditions are given for the existence of $k \leqq n^2$ conformal fundamental tensors. Next, expressions for the conformal curvature affinor in terms of the ordinary connection of K_n are derived. It is shown that: (1) if the unitary space K_n is conformal unitary euclidean, then the conformal curvature affinor vanishes; (2) no unitary space of constant nonvanishing curvature $K_n (n>2)$ is conformal to a unitary euclidean space. (Received January 6, 1942.)

127. Nathaniel Coburn: *Congruences in unitary space.*

The general properties of the congruence affinors of ∞^{n-1} curves, which are imbedded in a unitary space of n dimensions, K_n, are developed. The case in which the ∞^{n-1} congruence curves are either real curves X_1 or unitary curves U_1 is completely characterized. Next, by a study of two systems of Pfaffians, two types of orthogonality are defined: (1) ∞^1 hypersurfaces which are *completely unitary* orthogonal to the congruence curves; (2) ∞^1 hypersurfaces which are *semi-unitary* orthogonal to the congruence curves. It is shown that: (1) the ∞^1 completely unitary orthogonal hypersurfaces are ∞^1 unitary K_{n-1}; (2) the ∞^1 semi-unitary orthogonal hypersurfaces are ∞^1 semi-analytic spaces X_{n-1}. An additional analytical characterization of these two types of hypersurfaces is given. The final section of the paper is devoted to two problems: (1) the characterization in terms of congruence affinors of these two types of hypersurfaces; (2) special properties of congruences associated with each type of unitary orthogonal hypersurface. The desired characterizations are obtained in each case. Further, special properties of these congruences are found. Many of these properties are extensions of congruence theorems in Riemannian space. (Received January 6, 1942.)

128. J. J. DeCicco: *A generalization of the dual-isothermal transformations.*

In this paper is discussed a generalization in the plane of the theorem that the only lineal element transformations which carry every dual-isothermal family of curves into a dual-isothermal family are $U=\phi$, $V=(a_2v+b_2w+c_2)/(a_1v+b_1w+c_1)$, $W=(a_3v+b_3w+c_3)/(a_1v+b_1w+c_1)$, where ϕ, a_k, b_k, c_k are functions of u only. A field element may be defined by (u, v, w, p, q), where (u, v, w) are the hessian coordinates of the lineal elements, $p=w_u$, and $q=w_v$. All field element to lineal element transformations which carry every dual-isothermal family into a dual-isothermal family are $U=\phi$, $V=(a_2v+b_2p+c_2)/(a_1v+b_1p+c_1)$, $W=(a_3v+b_3p+c_3)/(a_1v+b_1p+c_1)$, where ϕ, a_k, b_k, c_k are functions of u, q, and $w-qv$ only. The contact field element transformations preserving the dual-isothermal character are also determined. (In the statement of this result, those transformations which carry every field into a single dual-isothermal field are excluded.) (Received December 4, 1941.)

129. J. J. DeCicco: *Point transformations by which straight lines correspond to circles.*

By means of some of Kasner's theorems (*The generalized Beltrami problem concerning geodesic representation*, Transactions of this Society, vol. 4 (1903), pp. 149–152), the author determines all the point transformations by which all the straight lines of the XY-plane correspond to circles of the xy-plane. These form the eleven-parameter set (not a group) $X=(a_2\{x^2+y^2\}+b_2x+c_2y+d_2)/(a_1\{x^2+y^2\}+b_1x+c_1y+d_1)$, $Y=(a_3\{x^2+y^2\}+b_3x+c_3y+d_3)/(a_1\{x^2+y^2\}+b_1x+c_1y+d_1)$. The family of circles consists of the ∞^2 circles orthogonal to a fixed circle (real or imaginary). Any transformation carrying more than $6\infty^1$ circles into straight lines must belong to our set. Conversely, any correspondence converting more than $6\infty^1$ straight lines into circles must be the inverse of a correspondence of our set. Obviously the set contains both the projective and the Moebius groups. (Received December 4, 1941.)

130. Jesse Douglas: *On the geodesic surfaces of a given curve family.*

Consider a family $\mathcal{J}$ of ∞^4 curves in space defined by differential equations of the form: $y''=F(x, y, z, y', z')$, $z''=G(x, y, z, y', z')$. Definitions: (1) relative to $\mathcal{J}$, a surface S is geodesic at one of its points p if every curve of $\mathcal{J}$ tangent to S at p lies entirely upon S; (2) a (totally) *geodesic surface* $\mathcal{G}$ is one which is geodesic at each of its points. In the general case, geodesic surfaces relative to $\mathcal{J}$ are non-existent. The following are shown to be the sole possibilities in which there is at least one geodesic surface through each curve C of $\mathcal{J}$: ∞^1 surfaces $\mathcal{G}$ through each curve C of $\mathcal{J}$ (*linear* family); exactly *two* surfaces $\mathcal{G}$ through each curve C (*intersectional* family, obtainable by cutting ∞^2 surfaces S with ∞^2 surfaces S'); exactly *one* surface $\mathcal{G}$ through each curve C (*semi-intersectional*). Examples of each of the preceding are easily given. Besides, the theory brings out the following possibilities: three or four surfaces $\mathcal{G}$ through each curve C, but the *effective* existence of these types seems doubtful. If n is a finite integer greater than 4, the type: n surfaces $\mathcal{G}$ through each curve C is definitely impossible. An important application is made to the author's work on the inverse problem of the calculus of variations: every curve family $\mathcal{J}$ of *intersectional* type is *extremal*, and in a highly general manner. (Received January 22, 1942.)

131. Edward Kasner and J. J. DeCicco: *A generalization of the isothermal transformations.*

In this paper is obtained a generalization of Kasner's theorem that the only lineal element transformations which carry every isothermal family into an isothermal family are $X+iY=F(x\pm iy)$, $\Theta=a\theta+h$, where a is a nonzero constant and h is a harmonic function. A field element (flat field) is defined by (x, y, θ, p, q), where (x, y) are the cartesian coordinates of the point, θ is the inclination, $p=\theta_x$, and $q=\theta_y$. There are seven types of field element to lineal element transformations which preserve the isothermal character. In the real domain, there are three types: (I) and (II) $X+iY=F(x\pm iy,\ p\mp iq)$, $\Theta=a\theta+h$, where h is a biharmonic function; and (III) $X=\phi$, $Y=\psi$, $\Theta=h(X,\ Y)$, where ϕ and ψ are arbitrary functions, and h is a harmonic function of $(X,\ Y)$. Within the group of contact field element transformations, there are seven types. In the real domain there are five types: (I) and (II) The Kasner extended group; (III) and (IV) $p-iq=F(X\pm iY,\ x+iy)$, $\Theta=a[\theta-2\int(pdx+qdy)]+h(X,\ Y)$; and (V) $X=\phi$, $Y=\psi$, $\Theta=h(X,\ Y)$. Special cases are that the isogonal, the multiplicative, and the isocline trajectories of an isothermal family are always isothermal. (Received December 4, 1941.)

132. Edward Kasner and Don Mittleman: *A general theorem on the initial curvatures of dynamical trajectories.*

The theorem proved is an extension of Kasner's dynamical theorem which states that: If a particle starts from rest in any positional field of force, the initial curvature of the trajectory is one-third of the curvature of the line of force through the initial position. The generalized result is: If a particle starts from maximum rest in an acceleration field of order n, the initial curvature of the trajectory is $n!(n-1)!/(2n-1)!$ of the curvature of the line of force through the initial position. This paper will appear in the Proceedings of the National Academy of Sciences. (Received January 19, 1942.)

133. W. H. Roever: *Geometric statement of a fundamental theorem for four-dimensional orthographic axonometry.*

The theorem stated below makes possible the construction of three-dimensional models for the "picturization" of four-dimensional space in a manner analogous to that furnished by three-dimensional orthographic axonometry for the picturization of three-dimensional space on the plane. Theorem: Three conjugate diameters and the axis of revolution of an oblate spheroid may be regarded, as far as directions are concerned, as the orthographic projection on the three-dimensional space, in which the spheroid lies, of four mutually perpendicular concurrent axes of four-dimensional space. If these axes be taken as rectangular cartesian axes and the three-dimensional space of the spheroid as the picture space, then the ratios which the halves of the chosen conjugate diameters and the radius of the spheroid's focal circle bear to the spheroid's equatorial radius are the foreshortening ratios, that is, the numbers by which the scales on the axes of the four-dimensional space must be multiplied respectively in order to obtain those on the axonometric axes (that is, the three conjugate diameters and the axis of revolution of the given spheroid). (Received December 8, 1941.)

Logic and Foundations

134. A. R. Schweitzer: *On a class of ordered $(n+1)$-ads relevant to the algebra of logic.* I.

Within the frame of his geometric theory (American Journal of Mathematics, vol. 31, (1909), pp. 365–410, chap. 2, 3) the author generates complete classes of ordered $(n+1)$-ads $(n=1, 2, 3, \cdots)$ of generalized type of constituents in the algebra of logic by means of two equivalent processes: (1) Relatively to the given ordered $(n+1)$-ad $\alpha_1\alpha_2\cdots\alpha_{n+1}$ and the ordered dyads $(\alpha_1\lambda_1), \cdots, (\alpha_{n+1}\lambda_{n+1})$ the α's are replaced by the corresponding λ's either singly or in combination yielding $2^{n+1}(n+1)$-ads. (2) Relatively to α_1, λ_1 the elements $\alpha_2\alpha_3\cdots\alpha_{n+1}$; $\lambda_2\lambda_3\cdots\lambda_{n+1}$ are successively