ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

1. A. A. Albert: *Quadratic forms permitting composition.*

It is proved that a quadratic form over a field F permits composition if and only if it is equivalent in F to the norm form xx^J of an alternative algebra over F such that $x+x^J$ and xx^J are in F. These norm forms are shown to be quadratic forms in 1, 2, 4 or 8 indeterminates, except for the diagonal norm forms (in 2^t indeterminates) of the purely inseparable fields of degree 2^t and exponent 2 over F of characteristic 2. The result has never before been obtained for fields of characteristic 2. Indeed it seems only to have been given completely for algebraically closed fields of characteristic not two. (Received October 20, 1941.)

2. S. P. Avann: *Lattices with arbitrary automorphism groups.*

There exists a partially ordered set having an arbitrary group as its group of automorphisms. Hence there exists a distributive lattice with a given automorphism group. A partially ordered set with cyclic group C_n has at least 3 conjugate sets of n elements for $n \leqq 7$ and at least 2 for $n > 7$. From these facts minimal partially ordered sets, lattices, and distributive lattices with cyclic automorphism groups can easily be obtained. (Received October 25, 1941.)

3. S. P. Avann: *The τ-function and its application to semi-modular lattices.*

Let $\tau[L]$, $\sigma[L]$ be the number of join irreducibles and the number of meet irreducibles, respectively, in a finite lattice L. In a non-distributive Birkhoff lower semi-modular (ξ'') lattice $\sigma > \tau \geqq \rho$, where ρ is rank of lattice. In a finite lattice L the following conditions are equivalent: (1) every element has unique irredundant join irreducible representation; (2) the sublattice generated by the elements covered by an element is a Boolean algebra, for each element of the lattice; (3) L is a finite Jordan-Dedekind chain lattice with $\tau = \rho$; (4) L is a ξ'' lattice with every modular sublattice distributive; (5) L is a ξ'' lattice with no modular non-distributive sublattice with coverings; (6) L is a ξ'' lattice with no modular non-distributive sublattice; (7) L is a ξ'' lattice with $\tau = \rho$. Although some of the equivalences are known, a new method of proof in contrast to combinatorial methods is emphasized. Note (6)$\rightarrow$(5) and (3)$\rightarrow$(7). (Received October 25, 1941.)

4. Reinhold Baer: *Inverses and zero-divisors.*

It may happen that an element in a ring is both a zero-divisor and an inverse, that it possesses a right-inverse but no left-inverse, and that it is neither a zero-divisor nor an inverse. Thus there arises the problem of finding conditions which assure the absence of these paradoxical phenomena; and it is the object of the present note to show that chain conditions on the ideals serve this purpose. The methods employed in this investigation allow the existence of universal units to be proved at the same time. (Received November 25, 1941.)

5. I. A. Barnett and C. W. Mendel: *A property of the traces of a square matrix.*

Let A be a square matrix having as elements n^2 independent variables and let $t_1, t_2, \cdots, t_n$ be the traces of $A, A^2, \cdots, A^n$, respectively. Then the jacobian of the t_i with respect to each set of n variables chosen from the original n^2 variables is never identically zero provided only that at least one of the variables is a diagonal element of A. (Received November 18, 1941.)

6. Salomon Bochner and R. S. Phillips: *Absolutely convergent Fourier series in noncommutative normed rings.*

This is a first attempt to extend Gelfand's analysis of normed rings, with $|x\cdot y| \leqq |x|\cdot|y|$, from the commutative to the noncommutative case. If R_1 is any (noncommutative) normed ring, if $G=\{m\}$ is any commutative index group, and if R is the normed ring of sequences $A=\{a_m\}$, $a_m \in R_1$, $m\in G$, with the norm $|A|=\sum_m|a_m|$ and the product $A\cdot B=\sum_p a_{m-p}b_p$, then an element A of R has a left-inverse (two-sided inverse) if and only if for each character $\chi(m)$ on G the element $\sum_m a_m\chi(m)$ has a left-inverse (two-sided inverse) in R_1. The familiar case of $\{m\}$ being the additive group of all integers can also be treated very easily by Wiener's original method. (Received November 22, 1941.)

7. R. H. Bruck: *Certain numerical invariants of polyadics.*

Let $x(i_1, i_2, \cdots, i_p)$, where each index i_α ranges over the natural numbers $1, 2, \cdots$, be a set X of indeterminates over a field K, and denote by X_r the subset of X obtained by restricting the indices to the range $1, 2, \cdots, r$. For each r let there be given a form $F_r(X)$ (with assigned coefficients in K) multilinear in each of the p indices $i_1, i_2, \cdots, i_p$ and containing only the indeterminates of X_r; for example, if $p=2$, $F_r(X)$ could be taken to be the determinant $|x_{ij}|$, $i, j=1, 2, \cdots, r$. The sequence of forms $F_1(X), F_2(X), \cdots$ may be called a rank sequence. Let $A(S_1, S_2, \cdots, S_p)$ be a "polyadic" with components in K, where each label S_j ranges over a finite set of fixed labels and has associated with it a prescribed type of linear transformation. If the identification $x(i_1, i_2, \cdots, i_p)=A(S_{i_1}, S_{i_2}, \cdots, S_{i_p})$ is made, then to each rank sequence corresponds a numerical invariant (or rank) of the polyadic, defined as the first r, possibly infinite, for which $F_r(X)=0$. The procedure is used to derive most of the known ranks of tensors, (ordinary) matrices and n-way matrices. (Received November 24, 1941.)

8. R. H. Bruck: *Isotopy of algebras with a principal unit.*

In the notation of A. A. Albert (abstract 47-9-331; also *Structure of Algebras*, pp. 9–13) the following, where U, V, W are nonsingular linear transformations, are three equivalent definitions of the isotopy of $\mathfrak{A}_0$ and $\mathfrak{A}$: (1) $R_x^0=UR_yW$, $y=x^V$; (2) $T_x^0=VT_zW$, $z=x^U$; (3) $x\circ y=(x^U\cdot y^V)^W$. The present study concerns regular algebras; for example, division algebras and linear closures of finite quasi-groups. $\mathfrak{A}$ is defined to be regular if it contains a left-regular element a and a right-regular element b; here for example, a is left-regular if the equation $ax=0$ implies $x=0$. $\mathfrak{A}$ has an isotope with principal unit if and only if $\mathfrak{A}$ is regular; the transformations which yield such isotopes are of form $U=PR_b^{-1}$, $V=PT_a^{-1}$, $W=P^{-1}$. If $\mathfrak{A}$, $\mathfrak{B}$ are isotopes with principal units and $\mathfrak{A}_1$ is an invariant subalgebra of $\mathfrak{A}$, then $\mathfrak{B}$ contains an invariant subalgebra isotopic to $\mathfrak{A}_1$. Every isotope of a simple algebra with principal unit is a regular simple algebra, but not conversely. In the present paper various applications are considered. It is noted that the isotopes of a division algebra are division algebras, and that a division algebra of order two is isotopic to a field. (Received November 24, 1941.)

9. I. S. Cohen: *A generalization of Macauley's theorem on unmixed ideals.*

Let $\mathfrak{o}$ be the ring of nonhomogeneous coordinates of the general point of an r-dimensional algebraic variety V over a ground field of characteristic zero. Let $\mathfrak{p}$ be a prime ideal in $\mathfrak{o}$, let $\mathfrak{J}=\mathfrak{o}_\mathfrak{p}$ be the quotient ring of $\mathfrak{p}$, and let $\eta_1, \cdots, \eta_s$ be s elements in $\mathfrak{J}$ such that the ideal $\mathfrak{A}=\mathfrak{J}\cdot(\eta_1, \cdots, \eta_s)$ is of dimension less than or equal to $r-s$. It is proved that if $\mathfrak{p}$ represents a simple subvariety of V, then $\mathfrak{A}$ is unmixed, of dimension $r-s$. This result, which is of a local character, implies the following theorem in the large: If an ideal $\mathfrak{B}=\mathfrak{o}\cdot(\zeta_1, \cdots, \zeta_s)$ in $\mathfrak{o}$ is of dimension less than or equal to $r-s$, then any imbedded prime ideal of $\mathfrak{B}$ represents a singular subvariety of V. In the case where V is a linear space (that is, $\mathfrak{o}$ a polynomial ring) these theorems reduce to theorems which have been proved by van der Waerden and Macauley, respectively. (Received November 21, 1941.)

10. R. P. Dilworth: *A theorem on lattice homomorphisms.*

The following theorem is proved: *Let S be a partially ordered set containing a complete, atomic, Boolean algebra B. Then there is an order homomorphism of S onto B leaving B invariant, preserving cross-cuts if they exist, and preserving distributive unions.* For the case in which B contains only two elements, this theorem gives most of the known lattice imbedding theorems. For B finite, this theorem is basic in the study of lattice homomorphisms. (Received October 24, 1941.)

11. R. P. Dilworth: *Dependence relations in a Birkhoff lattice.*

Let P be a set of elements. A relation $\perp$ between elements and subsets of P is a *dependence* relation if (1) $p\perp S+p$, (2) $p\perp S$ and $S\perp T$ implies $p\perp T$, (3) $p\perp S+p'$ implies either $p\perp S$ or $p'\perp S+p$. If P is the set of points of a Birkhoff lattice L it is well known that $p\perp S$ if and only if $p\subseteq \Sigma S$ defines a dependence relation with the above properties. Now let P_k be the elements in L of rank k. Define $r(x)=\rho(x)-k+1$. A subset S of P_k is *independent* if $r(\Sigma T)\geqq n(T)$ for every subset T of S. $n(T)$ denotes the number of elements in T. Define $p\perp S$ if and only if an independent subset T of S exists such that $p+T$ is not independent. It is shown that $\perp$ so defined satisfies (1), (2), and (3). Furthermore the independent sets of P_k are characterized in terms of the lattice structure. Also, a number of applications to imbedding problems are given. (Received October 24, 1941.)

12. Marshall Hall and R. P. Dilworth: *The imbedding problem for modular lattices.*

We prove the following: *There are modular lattices of every dimension greater than* *three which cannot be imbedded in a complemented modular lattice. Every modular lattice of dimension three or less is imbeddable.* The proofs rest on the fact that a modular lattice having but one independent modular functional can be imbedded in a complemented modular lattice only if it can be imbedded in a projective geometry. (Received November 24, 1941.)

13. B. W. Jones: *The number of classes in related genera of quadratic forms.*

For every prime p, there is associated with any genus g_1, of quadratic forms, another genus g_2. An upper limit for the number of classes in the genus of g_2 is given which depends on the number of classes in g, and the automorphs of the forms in the genus g_1. Under certain conditions the upper limit is reached. (Received October 6, 1941.)

14. Irving Kaplansky and O. F. G. Schilling: *Some remarks on relatively complete fields.*

In a previous paper of one of the authors, the following question arose: Under what circumstances can a field K be relatively complete in two inequivalent valuations? In this note it is shown that a necessary and sufficient condition is that K be separably algebraically closed, a result analogous to F. K. Schmidt's theorem on multiply complete fields. It is also proved that if a field K admits only cyclic extensions and is not relatively complete in any valuation, then no finite extension of K can be relatively complete. (Received November 10, 1941.)

15. C. C. MacDuffee: *On the composition of algebraic forms.*

If F is an algebraic field of degree n and class number h, every integral number of F corresponds to a matrix of the form $x_1S_1+x_2S_2+\cdots+x_nS_n$ under the regular representation, where S_i is an $n\times n$ matrix with rational integral elements. Furthermore, every ideal of the ith class corresponds under the Poincaré correspondence to a matrix of the form $x_1A_1^{(i)}+x_2A_2^{(i)}+\cdots+x_nA_n^{(i)}$, and conversely, every matrix of this form corresponds to an ideal of the ith class. The polynomials $f_1=|x_1S_1+\cdots+x_nS_n|$, $f_i=|x_1A_1^{(i)}+\cdots+x_nA_n^{(i)}|$ are n-ary n-ic forms which admit composition in the sense of Gauss, and under this operation form a group isomorphic with the class group, the norm form f_1 being the identity. (Received November 25, 1941.)

16. Saunders MacLane and O. F. G. Schilling: *Groups of algebras over an algebraic number field.*

Let K/F be the join $K'\cup K''$ of two normal subfields K' and K'' over an algebraic number field F. Suppose that S, S', S'' are the groups of algebras, prime to the discriminant of K/F, which are split by the fields K, K' and K'', respectively. This paper treats the question: When does the hypothesis $K=K'\cup K''$ imply the corresponding relation $S=S'\cup S''$ for the groups of algebras? It turns out that this relation will hold if and only if the Galois groups of K, K', K'' satisfy a certain condition on the distribution of the elements of maximal order. In particular, this condition always holds if K is abelian and in a number of other cases. (Received October 2, 1941.)

17. R. S. Pate: *Functional homomorphisms.* I. Preliminary report.

Two groups G and G' are considered for which a functional homomorphism $f(g)$

of G to G' may be defined such that (1) $f(g)$ is a subset of elements of G', (2) every element of G' occurs in some $f(g)$ and (3) $f(g_1)f(g_2)=f(g_1g_2)$. The set f^{-1} consists of all elements x of G such that $f(x)=f(g)$. The set $f^{-1}(1)$ is an invariant subgroup of G and $f^{-1}(g)$ a coset of $f^{-1}(1)$. The set of all distinct $f(g)$ is a group G'/f which is simply isomorphic to $G/f^{-1}(1)$. If $f(1)$ is an invariant subgroup of G' and $f(g)$ a coset of $f(1)$, $f(g)$ is the customary true homomorphism between G and G'. Certain sets of conditions on $f(g)$ which reduce a functional homomorphism to a true one are considered. A rigid group is one such that every functional homomorphism of any group to it is a true homomorphism. A group A is the product of two groups B and C if A can be constructed from B and C by the well known multiple homomorphism method. Certain theorems concerning functional homomorphisms, rigid groups and products are proved. (Received October 23, 1941.)

18. Samuel Perlis: *Normal bases of cyclic fields of prime-power degree.*

Let Z be a separable, normal extension of finite degree over a field F. If a quantity u and its conjugates form a basis of Z/F, u is said to generate a normal basis of Z/F, and it is known that such a basis always exists. The present paper considers the case in which Z/F is cyclic of prime-power degree $n=p^e$. It is shown that if p is the characteristic of F, u generates a normal basis of Z/F if and only if the trace of u in Z/F is not zero, and all such quantities are easily found in terms of the structure theory of these fields as developed by Albert. This theorem is false if the characteristic of F is not p. Parallel to Albert's theory, the paper next assumes that F contains p distinct roots of unity and again determines necessary and sufficient conditions. The methods used throughout are elementary and make no appeal to the theories of algebras or representations. (Received November 24, 1941.)

19. O. F. G. Schilling: *Normal extensions of relatively complete fields.*

Let F be a relatively complete field with perfect residue class field. The author discusses the Hilbert theory of infinite normal extensions K/F. It is shown in special cases that the algebraic structure of K/F depends essentially on the residue class field and the value group of K. The results of the Hilbert theory are then employed for the solution of existence problems of normal fields K/F with assigned algebraic and arithmetic properties. (Received October 8, 1941.)

20. M. F. Smiley and W. R. Transue: *Metric lattices as singular metric spaces.*

The fact that each pair of elements of a metric lattice (G. Birkhoff, *Lattice Theory*, American Mathematical Society Colloquium Publications, vol. 25, p. 41) which are not comparable together with their sum and product form a *pseudo-linear quadruple* (L. M. Blumenthal, *Distance Geometries*, University of Missouri Studies, vol. 13 (1938), p. 48) suggests that the presence of "sufficiently many" pseudo-linear quadruples in a metric space (M, δ) might ensure that lattice operations could be defined in M so that (M, δ) becomes a metric lattice $(M, \delta, <)$. In this note it is shown that, while this statement is not true, the additional assumption of a weak form of either of two five point transitivities of metric betweenness (Everett Pitcher and M. F. Smiley, *Transitivities of betweenness*, this Bulletin, abstract 47-5-196) suffices. (Received November 20, 1941.)

21. R. M. Thrall and C. J. Nesbitt: *On the modular representations of the symmetric group.*

Let S_m denote the symmetric group of degree m. If $m<2p$ the order $m!$ of S_m is divisible by only the first power of the prime number p. The general theory of modular representations is fairly well developed in such cases. Nakayama (*On some modular properties of irreducible representations of symmetric groups* I, Japanese Journal of Mathematics, vol. 17 (1940), pp. 89–108; II, ibid., pp. 411–423) has recently determined the irreducible modular characters for $m<2p$. In the present paper the requirement of irreducibility is relaxed. The structure of the regular representation is given in full. It is shown that there are only a finite number of inequivalent indecomposable modular representations and these are all determined. In the final sections specific matrix forms for these representations are computed for the case $m=p$. (Received November 21, 1941.)

22. T. L. Wade and R. H. Bruck: *The characteristic function of a 2p-tensor.*

Let $A_{(j)}^{(i)} \equiv A_{j_1 \cdots j_p}^{i_1 \cdots i_p}$, $p>1$, be an arbitrary tensor of the type indicated by its indices, and $\delta_{(j)}^{(i)}$ be the numerical tensor defined in abstract 47-5-184. The determinant of $\lambda\delta_{(j)}^{(i)} - A_{(j)}^{(i)}$, a monic polynomial $f(\lambda)$ of degree $N=n^p$ (n the dimension of the coordinate system), is an invariant, which may be termed the characteristic function of $A_{(j)}^{(i)}$. The set, C, consisting of the coefficients of $f(\lambda)$, yields not all, as in the case $p=1$, but only N out of a total of N^2-n^2+1 functionally independent absolute invariants of $A_{(j)}^{(i)}$ (abstract 47-9-335). The members of C are invariants of the connex $A_{(j)}^{(i)} X^{(j)} Y_{(i)}$, in which $X^{(j)}$ undergoes an arbitrary nonsingular linear transformation of N-dimensional space, and $Y_{(i)}$ the contragredient transformation. This property is in fact characteristic of those invariants of $A_{(j)}^{(i)}$ dependent upon the set C. In particular, $X^{(j)}$ may be of the form $x_{1|}^{j_1} \cdots x_{p|}^{j_p}$, where the p vectors $x_{\alpha|}^{i}$ undergo different transformations; similarly for $Y_{(i)}$, where $y_i^{\alpha|}$ is contragedient to $x_{\alpha|}^{j}$. For $A_{(j)}^{(i)}$ bisymmetric $f(\lambda)$ factors, the factors being the analogously defined characteristic functions of the simple algebras of which the algebra of bisymmetric tensors is the direct sum. (Received November 24, 1941.)

23. Morgan Ward: *Conditions for a lattice to be a Boolean algebra.*

Let S be a lattice in which if $a \supset c \supset b$, elements w and r exist such that $a \supset w \supset b$, $a \supset r \supset b$, $a = w \cup c$, $b = r \cap c$. Then if every element of S has a unique complement, S is a Boolean algebra. (Received October 22, 1941.)

24. P. M. Whitman: *Note on a certain representation of lattice elements.* Preliminary report.

In a finite lattice L, if a given element A covers exactly r elements, then A can be represented as the join of r or fewer join-irreducible elements. If this L is the lattice of all splittings of some lattice (abstract 46-9-442), then the number cannot be less than r. Under substantial restrictions, the same conclusions can be stated about the lattice of all colonies of cells (of algae, and so on), descended from a certain cell, which precede a chosen colony, where the cells reproduce by subdivision of one cell into two cells. (Received November 24, 1941.)

25. L. R. Wilcox: *Extensions of semi-modular lattices.* III.

The author's result (abstract 47-5-208) is extended to all complemented semi-modular lattices of dimension equal to or greater than 4. The following theorem is proved. Let L be left complemented (abstract 47-9-356) with the further property that $b, c \in L$, $bc \neq 0$ implies $(a+b)c = a+bc$ for $a \leqq c$; suppose also that there exists in L a chain of length 6. Then there exists a complemented modular lattice Λ containing L order-isomorphically and having the properties (a) $a \in L$, $a \neq 0$, $b \geqq a$ implies $b \in L$, and (b) for $a \in \Lambda$, $b \in L$, $a \leqq b$ there exists $c \in L$ such that c is a complement (in Λ) of a in b. Properties (a), (b) characterize Λ uniquely up to isomorphisms. This theorem is a lattice-theoretic generalization of well known imbeddings of affine and hyperbolic spaces into projective spaces. (Received November 24, 1941.)

26. Leonard Carlitz: *q-Bernoulli numbers and polynomials.*

Rational functions of q are defined by means of $q(qb+1)^m = b^m$ $(m>1)$, where after expansion b^m is replaced by b_m; "polynomials" are defined by $b_m(x) = \sum_{\alpha=0}^{m} C_{m,\alpha} q^{\alpha x}[x]^{m-\alpha} b_\alpha$, where $[x] = (q^x - 1)/(q-1)$. Many of the properties of the ordinary Bernoulli numbers and polynomials are readily extended to these quantities; in addition there are certain formulas in the generalized case that are not easily specialized to the case $q=1$. Among possible explicit formulas for b_m may be mentioned $b_m = \sum_{s=0}^{m} 1/[s+1] \sum_{\alpha=0}^{s} (-1)^\alpha \left[{s \atop \alpha}\right] q^{\alpha(\alpha+1-2s)/2} \cdot [\alpha]^m$, which leads at once to a generalized Staudt-Clausen theorem: $b_m = \sum_{s=2}^{m+1} N_s(q)/F_s(q)$ $(m>0)$, where $F_s(q)$ is the cyclotomic polynomial and $\deg N_s < \deg F_s$. (Received November 24, 1941.)

27. Joseph Lehner: *The Ramanujan identities and congruences for powers of eleven.* Preliminary report.

The author proves the existence of a "Ramanujan identity" for the modulus 11^α $(\alpha \geqq 1)$. For $\alpha = 1, 2$, this identity implies the Ramanujan conjecture: $p(n) \equiv 0 \pmod{11^\alpha}$ if $24n \equiv 1 \pmod{11^\alpha}$. The methods used are those of Rademacher's paper *The Ramanujan identities under modular substitution* (to be published in the American Journal of Mathematics). A modification of Hecke's T-operator is used. This operator is defined as follows: $U_{11}F(\tau) = \Sigma_\lambda F(\tau + \lambda/11)$, λ mod 11. If $F(\tau)$ is a modular function belonging to $\Gamma_0(121)$, that subgroup of the modular group defined by $c \equiv 0 \pmod{121}$, then $U_{11}F$ belongs to $\Gamma_0(11)$. Then it can be expressed as a polynomial in $A(\tau)$, $B(\tau)$, certain well known functions which constitute a basis for $\Gamma_0(11)$. By taking F to be $\eta(121\tau)/\eta(\tau)$, where $\eta(\tau)$ is the well known elliptic modular function of Dedekind, we obtain the desired Ramanujan identity for the modulus 11. Identities for higher powers of 11 are then obtained by a two-fold induction, one for even α, the other for odd α. The possibility of proving Ramanujan's conjecture for higher values of α $(\alpha > 2)$ is being investigated. (Received November 19, 1941.)

ANALYSIS

28. C. B. Barker: *The Lagrange multiplier rule for two dependent and two independent variables.*

Let $\bar{z}_1(x, y)$ and $\bar{z}_2(x, y)$ be of class C'''' on a closed simply connected region $\overline{G}$ of class C_α''' and minimize (1) $\iint_G f(x, y, z_1, z_2, p_1, p_2, q_1, q_2)dxdy$ among all pairs of functions $z_1(x, y)$ and $z_2(x, y)$ which coincide on the boundary G^* with $\bar{z}_1(x, y)$ and $\bar{z}_2(x, y)$, respectively, and which satisfy (2) $\phi(x, y, z_1, z_2, p_1, p_2, q_1, q_2) = 0$ on G; assume that f