lem is thereby reduced to one of the determination of branch points of a system of nonlinear equations by means of a characteristic value problem for a system of linearized equations. The relations between $\delta \boldsymbol{M}$, $\delta \boldsymbol{N}$ and $\delta \boldsymbol{r}$ are, for an elastic shell, those following from Hooke's law and from Navier's hypothesis. The complete system of scalar equations of this problem is obtained by a method analogous to that employed by the author in his simplified derivation of the equations for small displacements of thin shells (American Journal of Mathematics, vol. 63 (1941), pp. 177–184). (Received July 28, 1941.)

416. Alexander Weinstein: *On the buckling of a rectangular clamped plate compressed in one direction.*

This problem can be solved by an extension of a general method of reduction of eigenvalue problems (A. Weinstein, Mémorial des Sciences Mathématiques, vol. 88 (1937)). It can be linked to the corresponding problem for a supported plate by a sequence of intermediate differential problems of the fourth order which give lower bounds for the eigenvalues. It follows from this method that the lowest eigenvalue for a square plate of area π^2 is >10.0 while an upper bound 10.4 has been computed by J. L. Maulbetsch (Journal of Applied Mechanics, vol. 49 (1937)) who used the Rayleigh-Ritz method. The present method which gives definitely lower bounds for all eigenvalues differs essentially from previous formal procedures which could not establish such results. (Received July 18, 1941.)

Geometry

417. E. F. Allen: *On a triangle inscribed in a rectangular hyperbola.*

In the study of inversive geometry the following formulas: $z\bar{z}=a^2$, $a^2z+t_1t_2\bar{z}=a^2(t_1+t_2)$, $a^2z+t_1^2\bar{z}=2a^2t_1$, $\bar{p}z+p\bar{z}=2a^2$ are respectively the self-conjugate equation of a circle, the equation of a line through two points on the circle, the equation of the tangent line, and the equation of the polar line of the point p with respect to the circle, where $z=x+iy$, $\bar{z}=x-iy$, and i is defined by the equation $i^2=-1$. If a point in the xy-plane is designated by $z=x+ry$, $\bar{z}=x-ry$, and r is defined by the equation $r^2=+1$, the base $z\bar{z}=a^2$ is the rectangular hyperbola $x^2-y^2=a^2$. It is proved that the above formulas hold. They still hold if r is defined by $r^2=-k$ or $r^2=+k$, where k is a real number. For any triangle inscribed in a rectangular hyperbola there exists a nine-point hyperbola having many of the characteristics of the nine-point circle of a triangle. An anti-orthocentric group of triangles is defined and it is proved that the four triangles of the group have a common nine-point hyperbola. (Received July 17, 1941.)

418. E. F. Beckenbach: *On the analytic prolongation of a minimal surface.*

In extension of a known result concerning the interior behavior of minimal surfaces, it is shown that if a minimal surface is bounded in part by a plane curve and if the surface approaches the plane orthogonally, then the surface can be extended analytically across the plane and the plane is a plane of symmetry for the extended minimal surface. (Received August 2, 1941.)

419. Nathaniel Coburn: *Semi-analytic unitary subspaces of unitary spaces.*

Suppose a Hermitian space X_m of m-dimensions is imbedded in an n-dimensional unitary space K_n. By use of composite invariants of K_n, a theory of X_m in K_n is constructed which is a generalization of the known theory for K_m in K_n (analytic unitary subspaces in K_n). This X_m is invariant under the semi-analytic group of subspace coordinate transformations. By definition the X_m becomes an S_m (semi-analytic unitary subspace) when a set of preferred coordinate systems exist, related by a group of coordinate transformations (the group of S_m), for which certain components of the connection of X_m vanish. It is shown that the analytic group is a subgroup of the group of S_m which in turn is a subgroup of the semi-analytic group of X_m. The metric tensor of S_m is discussed and it is shown that if a coordinate system exists in S_m for which the ranks of certain matrices formed from the metric tensor of S_m are m, then S_m is a unitary euclidean space. This surprising result means that the conditions that an Hermitian X_m be an S_m are very restrictive. (Received June 2, 1941.)

420. J. J. DeCicco: *Geometric characterization of functions of n complex variables.*

In this paper Kasner's geometric characterization of the infinite group, consisting of two functions of two complex variables (Transactions of this Society, vol. 48 (1940), pp. 50–62), is generalized from four to $2n$-dimensional space S_{2n}. The infinite group of all correspondences in S_{2n} consisting of n functions of n complex variables has been termed the pseudo-conformal group G by Kasner. In the present paper it is proved that under G an r dimensional hypersurface and a $(2n-r)$ dimensional hypersurface possess at their point of intersection r (if $r \leqq n$) independent invariants. For any fixed value of r this characterizes G. Perhaps the simplest characterization (obtained as a corollary of the preceding by setting $r=1$) is that the pseudo-conformal group G of S_{2n} is defined by the preservation of the pseudo-angle between any curve C and any $(2n-1)$-dimensional hypersurface H at their common point of intersection. For $n=2$, this theorem becomes Kasner's characterization of the pseudo-conformal group in four dimensions. (Received July 21, 1941.)

421. J. H. Giese: *Conformally flat hypersurfaces of spaces of constant curvature.*

In this paper conformally flat n-dimensional $(n \geqq 4)$ riemannian spaces that can be imbedded isometrically locally in $(n+1)$-dimensional spaces of constant curvature are characterized by means of the characteristic roots and elementary divisors of the characteristic matrices of the metric and Ricci or second fundamental tensors. These spaces are intrinsically rigid in the enveloping space. Either all principal normal curvatures are equal, which for simple elementary divisors yields spaces of constant curvature, or else there is an $(n-1)$-fold principal normal curvature and a simple one. In the latter case there exists a preferred congruence of lines of curvature. This congruence is normal. The corresponding orthogonal $(n-1)$-dimensional hypersurfaces are of constant curvature and are mapped conformally upon one another by the preferred lines of curvature. The existence of such spaces is shown by constructing all suitable metric tensors $g_{ij}=e_i e^{2\sigma}\delta_{ij}$, each e_i being $+1$ or -1. These metrics are divided into $n+1$ inequivalent classes, n of which are proved non-null by Liouville's theorem on conformal mapping and by the groups of motions admitted by these g_{ij}'s. (Received July 17, 1941.)

422. F. E. Hohn: *On Cartan's projective involutes.*

In his *Leçons sur la Théorie des Espaces à Connexion Projective*, Cartan defines a projective involute of the curve (C), $A(t)$, to be a curve $P(t)$ such that for each t the tangent at P passes through A, and such that P lies on the osculating conic of (C) at A. If this last restriction be removed, the projective involutes of (C) are given by $P=(1+\theta')A+\theta A'+\theta^2A''$, where θ is an arbitrary function of t. Some of the systems obtained by putting conditions on θ' are studied, as is also the system of curves $P=A+h(c-t)A'+k(c-t)^2A''$. Various general cross-ratio properties are established, and Cartan's systems of involutes are developed as special cases. (Received July 28, 1941.)

423. T. R. Hollcroft: *Types of distinct double points of primals.*

There are r types of distinct simple double points of a primal f in S_r. A double point of type k, $1\leqq k\leqq r$, of f at a point P has a quadric tangent cone of species k whose vertex is an S_{k-1} tangent to f at P. For $k=1$, the double point is a node. For $1\leqq k\leqq r-2$, the cone is not composite. For $k=r-1$, the cone consists of two primes and the point is a binode. When $k=r$, the two primes coincide and the point is a unode. The number and forms of the characteristic conditions for each type of double point are obtained. Equations of primals of order n having any of the above types of double point are written. In particular, the following equation $[\sum i=1 \text{ to } r-k+1]\lambda_i\phi_i^2+[\sum j=r-k+2 \text{ to } r]\lambda_j\psi_j^3=0$, wherein ϕ and ψ are non-singular primals of orders 3α and 2α respectively, defines a system of primals of order 6α which has as basis points $2^{k-1}3^{r-k+1}\alpha^r$ double points of type k, $1\leqq k\leqq r$. (Received July 17, 1941.)

424. Edward Kasner and J. J. DeCicco: *Pseudo-conformal geometry: functions of two complex variables.*

Kasner has termed the infinite group G of all correspondences in four-dimensional space S_4, consisting of pairs of functions of two complex variables, the pseudo-conformal group G. He has proved that G is characterized by the preservation of the pseudo-angle between any curve C and any hypersurface H at their point of intersection (Transactions of this Society, vol. 48 (1940), pp. 50–62). In the present paper are found all the invariants of first order between the curves, surfaces, and hypersurfaces at a fixed point under G. A general pair of curve (hypersurface) elements possesses no invariants, whereas an isoclinal pair has a unique invariant. To any general surface element S there is associated a regulus R of curve elements. There are no invariants between a surface S and a curve element E (hypersurface element π) which is not on (not tangent to) the regulus R of S. On the other hand, if E (π) is in R (tangent to R), then there is a unique invariant between E (π) and S. Finally two general surface elements possess two independent invariants. (Received July 21, 1941.)

425. Marston Morse and G. A. Hedlund: *Manifolds without conjugate points.*

A directed geodesic g on a two-dimensional Riemannian manifold M is transitive if the elements (an element is a point and a direction at the point) on g are everywhere dense in the totality of elements on M. The existence of transitive geodesics has been proved under certain instability conditions by each of the authors. The hypotheses of all previous proofs have implied that no geodesic have on it two mutually conjugate points. The present paper proves that for a large class of two-dimensional manifolds the hypothesis that no geodesic have on it two mutually conjugate points is sufficient to imply the existence of transitive geodesics. This class includes, in particular, all closed, orientable, class C^3, two-dimensional Riemannian manifolds of genus greater than one. The implications of the non-existence of conjugate points are also investigated in the case of surfaces of the topological type of the torus. It is found that to a large extent the geodesics have the properties of those on a manifold for which the Gaussian curvature vanishes identically. If an added hypothesis concerning the non-existence of focal points is invoked, it can be shown that the curvature does vanish identically. (Received June 7, 1941.)

426. Abraham Schwartz: *A consequence of the Ricci equations for Riemannian manifolds.*

The paper considers the normal spaces at a point of a riemannian manifold of m dimensions which is imbedded in a manifold of constant curvature of n dimensions, $V_m \subset S_n$. With each normal space, $x=1, \cdots, k$, there is associated a curvature tensor $H(x)$ in a well known way, each tensor $H(x)$ being related to the preceding one, $H(x-1)$, by a Ricci equation. From these Ricci relationships information concerning the dimensionalities of the various normal spaces can be deduced. For instance, if the first normal space is one-dimensional, then the second normal space is zero or one-dimensional according as the rank of the second fundamental form is $\geqq 2$ or <2; if the first normal space is two-dimensional, then the second normal space is zero-, one-, or two-dimensional depending on the nature of the elementary divisors of the two second fundamental forms. Analogous theorems are true for the other normal spaces. (Received July 24, 1941.)

427. S. M. Ulam and D. H. Hyers: *On approximate isometries.* Preliminary report.

Let E and F be metric spaces and let ϵ be a positive number. The question studied is the following. If $T(x)$ is a continuous transformation of E into F such that $|\rho(T(x), T(y)) - \rho(x, y)| \leqq \epsilon$ for all x and y, then does there exist an isometric transformation $S(x)$ of E into F such that for all, x, $\rho(S(x), T(x)) \leqq k\epsilon$, where k is a positive constant, depending only on the spaces E and F? The question is answered in the affirmative for the cases in which E and F are Hilbert spaces or finitely dimensional euclidean spaces. (Received July 30, 1941.)

Logic and Foundations

428. Garrett Birkhoff: *Metric foundations of geometry.*

New derivations of fundamental theorems of euclidean, hyperbolic and spherical geometry, with particular reference to the lattice of subspaces, are given. The postulates are: (1) metric postulates of Fréchet, (2) straight line postulates of Menger, in a weakened form valid in any riemannian geometry, (3) the postulate of Pasch. Thus the local compactness postulate of Busemann's system is dispensed with, and his straight line postulates are weakened; on the other hand, the postulate of Pasch is not restricted to 3 points. The proofs are elementary throughout. (Received July 7, 1941.)

429. Henry Blumberg: *A reconsideration of the paradoxes in the theory of sets.*

Proceeding from an intuitive bias that it should be possible to eliminate the paradoxes in question along the line of natural human understanding, without reforming our familiar logic or invoking ingenious technical or professional stratagems, the