calculus of binary relations in terms of the two operations $|$ and $<$. In his paper McKinsey shows that $<$ is definable in terms of $|$ but not conversely. In this paper the author develops a set of independent postulates for the calculus of binary relations in terms of the single operation $|$. (Received January 25, 1941.)

Statistics and Probability

169. Archie Blake: *The exploratory determination of statistical distributions.*

The most powerful statistics are not always the most "efficient" or those whose distributions have already been tabled, but their distributions can be computed in small samples without inordinate labor. To find the distribution of $g(X=x_1, \cdots, x_n)$ subject to the condition $P(X)$, compute requisite values of g^{-1} (multiple-valued) and require $\int^{P,g^{-1}} f(X)dX$, f being the given distribution function. The chief task is the computation of many values of the functions involved; this is alleviated by modern machine methods (especially punched cards). Tables of f, g, and so on, with their derivatives or the required fractions of the latter are prepared once for all on cards; thereafter the work consists only of interpolation. (Taylor's series recommends itself in this problem, as it converges more rapidly than ordinary interpolation formulae, and in the case of multivariate interpolation is much less complicated.) For a statistic whose asymptotic distribution is known, we can interpolate approximately between this and the results of the computation for small n to obtain an estimated distribution for any n. (Received January 23, 1941.)

170. W. G. Madow: *The distribution of the general quadratic form in normally distributed random variables.*

The distribution of the general quadratic form in normally distributed random variables is obtained. This distribution is used to obtain the distribution of Neyman's estimate in the theory of the representative method of sampling, and it is also used to obtain a generalization of P. L. Hsu's distribution of Student's ratio when the true means and variances are unequal. The distribution is also used in tests occurring in the analysis of variance with non-orthogonal data, and the study of differences of various orders. In the latter use, a test for periodicity is obtained. (Received January 25, 1941.)

171. Henry Scheffé: *An inverse problem in correlation theory.*

An $m\times n$ matrix Y may be used to represent m sets of measurements on n variables. The $n\times n$ matrix R of correlation coefficients r_{ij} is a function of the matrix Y, $R=F(Y)$. Necessary conditions (C) that $R=F(Y)$ are that R be real symmetric with diagonal elements unity, and positive (rank=index). Given any matrix R satisfying the conditions (C), does a "statistics problem Y" exist such that $R=F(Y)$? It is proved by matrix methods that there are no solutions Y with $m\leqq$rank R, but ∞ solutions for each $m>$rank R. Particular solutions are constructed and the most general solution is characterized. Some corollaries are drawn. (Received January 8, 1941.)

172. Jacob Wolfowitz: *Tests of statistical hypotheses where the distribution forms are unknown.*

The likelihood ratio criterion for testing composite statistical hypotheses, discovered by Neyman and Pearson and recently proved by Wald to be asymptotically most powerful, is extended to testing composite hypotheses where the forms of the distribution functions are entirely unknown (continuity is assumed) and where tests must be based on the order relations among the observations. Thus a general method for treating problems of this character is obtained. For the problem of two samples (Wald and Wolfowitz, Annals of Mathematical Statistics, June, 1940) the resultant statistic is $\prod_j l_j^{l_j}/(l_j)!$, where l_j is the length of the jth run. Its logarithm is asymptotically normally distributed. The result is immediately extensible to the problem of k samples. For the problem of independence (Hotelling and Pabst, Annals of Mathematical Statistics, March, 1936) a similar statistic is obtainable which differs from the commonly used rank correlation coefficient. The method used to prove the logarithms of these statistics asymptotically normally distributed is applicable to proving the asymptotic normality of a large class of functions of partitions of an integer, of functions of sequences where the subsequences of odd and even numbered elements are themselves partitions of different integers of fixed ratio, and to similar problems (Received January 13, 1941.)

Theory of Numbers

173. Paul Erdös and Joseph Lehner: *On the distribution of the number of summands in the partitions of a positive integer.*

Let $p_k(n)$ denote the number of partitions of n into not more than k summands. Then for $k=n^{1/2}(\log n/C)+xn^{1/2}$, $C=\pi(2/3)^{1/2}$, $p_k(n)/p(n)\sim\exp\{-2\exp(-Cx/2)/C\}$, where $p(n)$ is the number of unrestricted partitions of n. For $k=o(n^{1/3})$, $p_k(n) \sim C_{n-1,k-1}/k!$, uniformly in k. Let $P(n)$ be the number of partitions of n into different summands. Then for "almost all" partitions, the number of summands in a given partition not exceeding $xn^{1/2}$ lies between $2n^{1/2}\{\log 2/(1+\exp(-Dx))\}/D\pm\epsilon n^{1/2}$, $D=\pi(1/3)^{1/2}$. The methods used are elementary in character. (Received January 23, 1941.)

174. Gordon Pall: *The construction of positive ternary quadratic forms.*

A method is developed of writing down quickly the reduced integral positive ternary quadratic forms of a given determinant, order, or genus. (Received January 30, 1941.)

175. H. A. Rademacher: *Ramanujan's identities under modular transformations.*

If the Ramanujan identities which exhibit the divisibility of $p(5n+4)$ and $p(7n+5)$ by 5 and 7, respectively, are expressed in terms of the Dedekind function $\eta(\tau)$, they can be subjected to modular transformations. Each of the two identities goes over into a new one, which is noteworthy because of the occurrence of the Legendre residue symbol. These new identities lead to a direct proof of the Ramanujan identities. The same procedure can be applied to an identity given by Zuckerman, involving $p(13n+6)$. Other identities, proved by Watson and Zuckerman, lead to modular equations of "level" (Stufe) 5 and 7. (Received January 24, 1941.)

Topology

176. F. B. Jones: *Monotonic collections of peripherally-separable connected domains.*

It is shown that: *In a locally connected metric space, every monotonic collection of*