as solutions of Bessel's differential equation. Our author, however, follows a different path. His starting point is the wave equation, from which he derives first plane waves, and then, by superposition of these, spherical and cylindrical waves. This leads immediately to the Sommerfeld integral representation of the Hankel functions, which then serve as the basis for defining the J and N functions and for deriving the important properties of the cylinder functions.

The author discusses the following aspects of cylinder functions: power series, asymptotic expansions of Hankel and Debye, various integral representations, recurrence relations, zeros, definite and indefinite integrals, boundary value problems and applications. It will be noted that the author has succeeded in covering the most important topics in a remarkably small number of pages. But the value of the book must not be judged by its brevity. It contains a carefully planned exposition of the theory and will serve as a valuable study and reference book.

C. A. Shook

Differentialgeometrie der Kurven und Flächen und Tensorrechnung. By Václav Hlavatý. Groningen, Noordhoff, 1939. 11+569 pp.

This treatise presents a large portion of the classical differential geometry of one and two dimensional subspaces of ordinary euclidean space. Just enough vector and tensor analysis is given to enable the reader to manage profitably the abbreviated symbolism. Definitions and results are stated in such a way as to generalize readily to higher dimensions.

The first chapter is devoted to curves. After the theory is developed in terms of a general parameter, an account is given of the various specializations arising from the use of the arc length as parameter. In particular, the construction of a curve from its curvature and torsion is treated carefully.

The second chapter concerns those properties of a surface which depend only on its metric tensor. The absolute differential is used systematically. There is an unusually full discussion of the applicability of surfaces, including explicit equations for developing a surface on a plane or on a surface of revolution.

The normal to a surface leads, on differentiation, to a tensor associated with the behavior of the surface toward the ambient space. In the third chapter, those properties are discussed which depend on this (second fundamental) tensor. A feature of this chapter is the discussion of the explicit construction of surfaces having prescribed first or first and second fundamental tensors.

The fourth chapter contains applications to a variety of special surfaces, notably ruled surfaces and minimal surfaces. There is no treatment of quadrics nor of congruences of lines.

The book is carefully written, and a high level of explicitness is maintained in the details of the argument and in the treatment of special cases. The reviewer feels that two exceptions to this statement may confuse readers unfamiliar with tensor analysis. "Tensor" is so defined that there is no distinction between a tensor and the set of its components in any particular coordinate system. This distinction—analogous to that between the number two and a couple of apples—is the key to the geometric significance of tensors, and this point seems to have been obscured by the form given to the definition.

In the second place, "differential invariant" is defined in the algebraic sense—as a function which is transformed by substituting for the old variables the appropriate functions of the new variables and multiplying by a certain power of the Jacobian. It is then stated that "The problem of metric differential geometry . . . consists in the study of the invariants . . . which can be obtained from the equations of the surface (or curve)." As examples, two invariants for each dimension are constructed, the whole treatment covering about five pages. No further mention is made of the concept. This procedure appears open to several objections. First, no explicit connection is established between the remainder of the text and "the problem of differential geometry." Second, it seems pedagogically unsound to introduce such a basic concept without giving the reader more definite occasion to become familiar with its applications and to grasp at least one precise significance of a word having a confusing variety of meanings. Finally, the term has been so used in other books that tensors are "invariants." If a practice opposed to this usage is adopted, a word of caution to readers seems desirable.

The book has an ingenious index which greatly enhances its usefulness as a source for reference. There is no bibliography, nor exact reference to sources, nor exercises. In dealing with surfaces, roman numerals I and II are used as indices for quantities pertaining to the surface; the gain in emphasis seems offset by the visual hesitation in distinguishing I II from II I.

Chiefly for its thoroughgoing use of tensor methods, the book is a valuable complement to the available treatises. It is distinguished from them by numerous interesting details in the presentation.

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