

ries rather than individual theorems, the state of affairs in the theory of algebras permitting this luxury. Such an attitude is not possible with respect to groups of linear transformations; the essential reason is perhaps that algebras have been studied as representatives of a few large types, whereas groups are in many cases individuals or representatives of small families. Correspondingly, the essentials of the theory which Deuring covers are contained in comparatively few papers, whereas van der Waerden had to incorporate explicitly a host of individual investigations. If one should examine the *Fortschritte der Mathematik*, say since 1900, one would hardly find a paper related to the subject not mentioned in his book (excepting papers on continuous groups which were reserved for another report in the same collection).

Whatever the abstract algebraic method can do in the line of unification has been done, as in the theory of the classical linear groups (like the orthogonal groups, and so on), or the theory of representations of general classes like hypercomplex systems, finite groups, bounded representations of arbitrary groups; but in fields like groups of given degree (2, 3, 4), or the representation of individual groups (like modular groups), the special character of the problem necessitates a special treatment.

MAX ZORN

Mécanique Statistique Quantique. By Francis Perrin. (Traité du Calcul des Probabilités et de ses Applications, vol. 2, no. 5.) Paris, Gauthier-Villars, 1939. 224 pp.

This book forms a complete introductory outline of modern statistical mechanics. The first part (five chapters) deals with the rudimentary notions of classical statistical mechanics of Hamiltonian systems, ergodic theory, canonical ensembles, equipartition, coupled systems, thermostats, the thermodynamic quantities and laws, with applications to perfect gases and radiation. The second part (four chapters) introduces the rudiments of the quantum theory and extends to systems obeying its laws many of the considerations of the first part. The third part (entitled *Statistique Quantique des Systèmes Indiscernables*, eight chapters) forms the main body of the work, to which the earlier parts form a sort of introduction. The principles of indistinguishability and exclusivity are introduced and the Bose-Einstein and Fermi-Dirac schemata established. The three laws of thermodynamics are then derived from the statistical theory, and applications are made to the chemical constants, gaseous degeneres-

cence, magnetism, neutron stars, and so on. There follows a chapter on statistical kinetics in which the increase of entropy and tendency towards statistical equilibrium are treated. The closing chapter deals with indistinguishability and exclusivity from the viewpoint of symmetric and antisymmetric wave functions.

The author appears to have imposed two restrictions upon himself: *Firstly*, the mathematics used is elementary (advanced calculus, elementary combinatorial analysis, and Stirling's formula). The result is not merely that he exerts to the full the physicists' privilege of lack of rigor, but rather that, owing to the lack of the universal language and framework of conceptions with which modern mathematics would have supplied him, certain subjects are not placed in the clearest light. Thus, to give only one example, in the ergodic theory the author assumes all limiting functions to be continuous and hence loses sight of the nature of the present-day problem, which is one of measurable functions. *Secondly*, the quantum mechanics used is quite rudimentary, consisting solely of the simplest notions of the wave equation, quantized orbits and the principles of indistinguishability and exclusivity. Thus the great and fundamental questions (such as are treated in the last chapters of J. von Neumann's *Mathematische Grundlagen der Quantenmechanik*, and other modern works) which are suggested by the very title of the present book are left out of consideration. For the most part the method is the enumeration of possible distributions among states, the approximation by Stirling's formula and the maximizing of the resulting expression for the number of complexions, it being assumed that the course of the system is through conditions of maximum number of complexions. The justification of this in the chapter on statistical equilibrium is logically rather meagre. Granted these limitations, we feel that the author has been most successful.

One point deserves especial commendation: The author defines in the following precise mathematical form the idea of two ergodic systems (that is, closed metrically transitive hamiltonian systems) "in feeble interaction": (1) for all calculations for finite time, the combined equation of motion is to be taken the same as the combination of the two separate equations of motion (that is, without mutual potential term); (2) on the other hand, in limits ($t \rightarrow \infty$), the combined system is to be regarded as metrically transitive (that is, $H_1 + H_2 = E$ is to be regarded as the only integral, not $H_1 = E_1$ and $H_2 = E_2$). The intuitive notion of feeble interaction is familiar but, to our knowledge, it has not been given in such a clean-cut mathematical form.

We are of the opinion that the very fact which forms the strength of

this book is also the source of its weakness, namely, that it is in last analysis only an introductory outline. It is written with the traditional French clarity, and will be a useful addition to the library of the mathematician taking an interest in modern physics.

B. O. KOOPMAN

Topological Groups. By L. Pontrjagin. Translated by Emma Lehmer. Princeton, University Press, 1939. 9+299 pp.

The topological group is a combination of two fundamental mathematical concepts—group and topological space. A topological group G is a group and at the same time a topological space in which the group operations in G are continuous. Historically, the concept arose from the study of groups of continuous transformations. Pontrjagin gives an axiomatic treatment of topological groups. Later he points out their connections with continuous transformations as well as with other older concepts. In the language of the author: "This book is intended for the reader with rather modest mathematical preparation." This is accomplished very successfully by both the included material and its organization. All material needed is precisely formulated, and in most cases proofs are given. The understanding of the text is enhanced by the inclusion of seventy-five examples, which deal largely with real numbers, matrices, and vector spaces. The author points out questions left unanswered in most of the general problems discussed.

The first three chapters give an excellent introduction to topological groups. Chapter I discusses the usual topics in elementary abstract group theory. These include normal subgroups, factor groups, homomorphisms, the center of a group, direct products, and commutative groups. In Chapter II a topological space is defined by means of axioms in terms of the closure of a set. An equivalent neighborhood definition is set up and is used extensively. Among the concepts studied are connectedness, regularity, second axiom of countability, compactness, and topological products. Continuous mappings are introduced early and have a prominent place in the chapter. Chapter III contains the first steps in the theory of topological groups. The fundamental relations holding for abstract groups and topological spaces are adapted to topological groups. Additional concepts introduced include interior (open) mappings and local isomorphisms.

After the introductory material in the first three chapters, the reader may proceed to any one of Chapters IV, VI, or VIII. Chapter IV proves that any compact group satisfying the second axiom of