conformal language and notation. There is a certain finality about the content which makes it the foundation of every further study on the subject.

It is not always easy to follow the author and we would like to ask him to be kind with his readers when he gives the final touches to the other two volumes. As an example, let us take the beginning of the book. On page 1 we hurry immediately into the midst of things. An oriented sphere is defined by 5 pentaspherical coordinates connected by a quadratic relation. The meaning of these coordinates is not explained, we must take this from other books, and the quadratic relation is only given as $(yy)_5=1$, which leaves it to the reader to discover what it means. We are not informed either, whether the coordinates are ordinary or general pentaspherical coordinates, and have to discover this later, from the context. On page 2 we read that the different projective orientation processes must be understood in the sense of the author's non-euclidean geometry. This is an essential point, and we must therefore first go to volume 26 of the Tôhoku Mathematical Journal (1926) to find what it means. The same thing happens on page 3, where we are referred to another paper to find the meaning of certain equations expressing a doubly oriented sphere. Such difficulties could easily be avoided if the author, at the beginning, would not presuppose more than an average college knowledge of pentaspherical coordinates, projective and non-euclidean geometry, as, for example, Blaschke, Thomsen, or Fubini-Cech have done in their related expositions. We are sure that Professor Takasu will only do justice to his beautiful investigations if he can agree to such modifications in his presentation.

D. J. STRUIK

Superficie Razionali. By Fabio Conforto. Bologna, Zanichelli, 1939. 16+549 pp.

Although there is an extensive literature on rational surfaces, it is scattered through the periodicals in various languages, and the methods of proof differ widely as the theory gradually develops through more than a century. On the other hand, a knowledge of this field is indispensable to the study of algebraic geometry of more than two dimensions, and to some phases of analysis.

The purpose of the present book is to supply this need of a systematic development of the subject from the present point of view, starting at the beginning and providing all the necessary details of the general theory, but referring to original papers for further perusal of particular features. A knowledge of plane geometry, including plane Cremona transformations and algebraic plane curves is presupposed. Throughout the book the guiding principle is that of continuity of the parameters in a linear system, the methods of proof following closely those of the four volume treatise of Enriques and Chisisni[1] to which frequent reference is made.

The volume is divided into two parts, the first being concerned with rational surfaces of order less than 5, developed largely along traditional lines, and the other to the systematic theory. But this first part includes one feature that deserves special mention. After discussing those quartic surfaces that have a double curve, the book provides an exhaustive analysis of all the quartic surfaces that are rational and have just one isolated singular point. Here, in particular, the principle of continuity of the parameter in a linear system is employed in such a way that a limiting value of the parameter defines a surface with a singularity of the desired form. It is clearly shown that the threee types of Noether quartics correspond to the three types of plane involutions of order two, the images, on the surface, of a pair of conjugates on the plane, being a pair of residual points on a line through the double point. Other features in this first part are the exhaustive discussion of all possible cases, including the Steiner surface and Cayley's cubic scroll.

The second part begins with surfaces having a pencil of rational curves, then those having only rational plane sections, followed by elliptic, hyperelliptic ($p=2$) and those of genus 3, not hyperelliptic. In each case, the characteristic properties of the representative net of plane curves are established. Apparent digressions are the classifications of plane involutions of order two and the conditions for rationality of a double plane, but both results are shown to be necessary to insure that the classification of rational surfaces is complete.

A long chapter is devoted to other rational surfaces; it includes the quintics having a multiple curve, but does not claim to be exhaustive. The last chapter contains a complete treatment of the theorem of Castelnuovo that all plane involutions, of any order n, are rational. In the main, the proof in the original paper is reproduced, but important simplifications are achieved by systematic use of continuity in linear systems.

Frequent and extensive references to the literature are made, and a convenient index is provided, but much of the recent literature is not included, especially that of non-European origin; moreover, many

[1] *Lezioni sulla Teoria Geometrica delle Equazioni e delle Funzioni Algebriche*, Bologna, Zanichelli, 1915, 1918, 1923, 1934.

of the earlier theorems are not referred to their sources, but to later proofs of them given in works supposed to be more accessible.

The writing and proofreading have been done with great care; apart from a few trivial errors in orthography, the only error is the reference to G. Kantor, footnote on page 131, instead of S. Kantor.

This book will be welcomed by workers in algebraic geometry; it competently fills a gap in the preparatory literature.

VIRGIL SNYDER

Modern Science, A Study of Physical Science in the World Today. By Hyman Levy. New York, Knopf, 1939. 736 pp. 160 illustrations.

Professor Levy's ability to present general relationships and abstractions in an interesting simple way has been commended by reviewers of his previous recent book "A Philosophy for a Modern Man." This faculty is again evident in his analysis and evaluation of social and intellectual forces in the development of physical science.

Part I of the book deals with the background of social life within which science has developed as one among many "channels" of human energy. In an interesting discussion Professor Levy contends that the work of men like Newton was largely a consequence of commercial and other social factors rather than a spontaneous intellectual activity.

Part II is concerned with the nature, methods and unity of science. Considerable stress is laid on the common occurrence of sequences of phases separated by discontinuities in physical processes. Many usually overlooked examples of these phases are given.

Parts III, IV and V deal with mathematical symbols and physical (theoretical) models. Some general algebraic and geometrical concepts are discussed. The idea of limits is presented in a remarkably concrete way. The rigor is surprisingly good for a popular discussion, but there are some undesirable implications, for example, termwise differentiability of infinite series is accepted without question. An instructive treatment of mass, momentum, impulse and energy is given. Aeronautical science is used to illustrate the unity of theory and experiment. The development of non-euclidean geometries is used to illustrate how science can "shake off the past."

Part VI is rather discursive in its discussion of astronomy, geology, atomic theory and relativity. The treatment of mountain building does not include the more modern theories of Joly and others.

Part VII on the "Age of Light" uses the history of methods of illumination and their social consequences to illustrate the depend-