of 3-space, or a sheaf of curves obtained similarly from a bundle of parallel lines. Other higher dimensional analogues are evident, but these two seem to be the only ones whose properties have been studied to any great extent.

The first part of the book is devoted mainly to "hexagonal" webs, in which there exist certain configurations formed by the curves of the web (analogous to the configurations of Desargues or Pappus in projective geometry). The principal result is the following:

Any hexagonal n-web can be mapped onto n pencils of lines in a plane if $n \neq 5$. *There exist 5-webs which cannot be so mapped, but these can be mapped onto four pencils of lines and the pencil of conics on their centers.*

This theorem is proved for $n=3$ (the first non-trivial case) by topological methods, and then extended to higher values of n by the use of properties of continuous groups. Similar theorems are obtained for the two 3-dimensional webs mentioned above.

In the remaining two parts of the book the discussion is limited to those webs and transformations for which the defining functions have a suitable number of derivatives. Part II treats of the differential invariants of webs; including among other topics the determination of complete sets of invariants, a characterization of hexagonal webs, and the relation between webs and the theory of parallelism in differential geometry. Extensive use is made here of the properties of Lie groups and the differential operators arising from them. In Part III webs are discussed from the point of view of algebraic geometry, the main tools being Abel's theorem and the properties of Abelian integrals. The *rank* of a web is introduced as the number of ways of setting up certain parametrizations of the curves of the web. The principal theorems in this connection are the following:

If an n-web of lines in a plane has positive rank, it consists of tangents to an algebraic curve (possibly reducible) of class n.

Any 3- or 4-web of maximal rank can be mapped on a web of lines in a plane.

Probably the most striking feature of the book is the way in which so many different branches of geometry are applied to the development of the theory of webs. There are numerous applications of theorems in topology, continuous groups, algebraic geometry, and differential geometry. These theorems are not merely referred to, but proofs of them are given, at least for the special cases which are to be used. As a result the book is almost wholly self-contained, in spite of the wide range of the topics that are touched upon. This feature, and the inclusion of numerous exercises which supplement and extend the expounded material, aid in making the book an excellent introduction to an interesting new chapter in geometry.

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Analyse Mathématique. Vol. 2. *Équations Différentielles. Développements en Séries. Nombres Complexes. Intégrales Multiples.* By P. Appell. 5th edition completely revised. (Cours de Mathématiques Générales.) Paris, Gauthier-Villars, 1938. 305 pp.

The author devotes this second volume of his set on Mathematical Analysis to the following topics in the order mentioned: Differential Equations (91 pages, 3 chapters); Developments in Series (80 pages, 2 chapters); Complex Numbers (39 pages, 1 chapter); Multiple Integrals (78 pages, 2 chapters). The written text and the two appended notes are supplemented with 79 well-drawn figures and are divided into 177 sections each with a title and number. A table of contents, but no index, is bound with this second volume. No lists of exercises are included. However the various topics are illustrated by examples worked out by the author and these examples are liberally scattered throughout the volume. In fact not fewer than 14 of the numbered sections have the word "exemples" or "exemple" as a part or all of their titles.

The usual material on first order differential equations is given ending with a 10 page brief discussion of the general theory. To treat second order equations the author considers first those that are reducible to first order equations, next linear equations, next differential systems of the second order and then gives a few remarks about systems of n first order equations.

After deriving and illustrating Taylor's formula, applications are made to calculating approximate function values and to finding limits of indeterminate forms and order of contact of two curves. Infinite series are introduced, and in turn, power series. Application is made to the calculation of function values, to integration taking the rectification of the ellipse as an example, and to solving differential equations.

Defining a complex number as a number pair (a,b) rules of operation are developed and graphically illustrated. De Moivre's formula follows and then the theory developed is applied to solving second and third degree polynomial equations. Series of complex terms are introduced, e^z, $\sin z$, and $\cos z$ are defined and Euler's formulas obtained. Some applications are made to second order differential equations.

The double integral is defined as a limit and this same limit is shown to be a volume. It is shown to equal the iterated integral and its calculation illustrated in polar and rectangular coordinates. Areas of curved surfaces, infinite domains of integration and application of double integrals to simple integrals containing a parameter are taken up. Riemann's formula connecting a double integral and curvilinear integral is then dealt with briefly. The triple integral is defined as a limit of a sum, analogously to the double integral, and is applied to potentials, to centers of gravity, moments of inertia and surface integrals. A 10 page section on elements of vectorial analysis, including mention of Green's theorem and the formulas of Stokes and Ampère, concludes the volume.

A 7 page appended note gives an interesting discussion of the theory of mechanical integrators. A second 4 page note is given to developing and illustrating L'Hospital's rule.

To the reviewer the volume seems to be carefully written and readable. Only a few minor, but obvious, misprints were noticed.

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