for the parameters, their weights and errors, the adjustment of the observations, and the confidence belts associated with the curve.

Lastly there are interesting exercises and notes on the formation of the normal equations for various functions to best fit certain measurements such as straight line, parabola, exponential, exponential with a lineal component, the generalized hyperbola together with three examples in curve fitting completely worked out, viz.:

Example 1, fitting an isotherm $y=a+bx+cx^2+dx^4$ with parameters subject to the condition $x=1$ when $y=1$.

Example 2, the polynomial $y=a+bx+cx^2$ with both x and y subject to error.

Example 3, an example useful in forestry, fitting $x=ay^bz^c$ where x = volume of a tree, y = merchantable height of tree, and z = diameter at breast height, the data having been secured from 66 trees.

This treatise is to be commended for its completeness and for maintaining the single purpose of all work in least squares, the minimizing of $\sum \overline{\text{res}}^2$. Although the whole scheme is non-rigorous except for linear functions, the error introduced by ignoring the higher powers of the corrections to the assumed values for the parameters or other unknowns does not seriously affect the result. It seems to be the basis for a very good graduate course and shows clearly how the use of Lagrange multipliers may sometimes clarify an otherwise hopeless analytical problem.

JOHN H. OGBURN

Les Definitions Modernes de la Dimension. By Georges Bouligand. Paris, Hermann, 1935. 44 pp.

This volume opens with a brief discussion of linear or affine geometry based on postulating the existence of an abstract system of elements called vectors which combine amongst themselves and with the real numbers to give us new elements of the same class. The dimension is determined by the number of linearly independent vectors. There is a brief discussion of spaces which in this sense would be of infinitely many dimensions.

Then denying the right of geometry to limit itself merely to spaces of this category, the author discusses three types of definition for dimension:

(1) The definitions which follow the direction of Fréchet who is interested in the spaces introduced by the needs of General Analysis. Fréchet develops a theory of dimension types or a topological frame based on the notion of homeomorphism between sets. His theory has as its axiomatic substructure three simple axioms concerning the closure of point sets.

(2) Those definitions where the fundamental notion is that of *separation* of sets by various subsets. These definitions emanate from the influence of Poincaré who started from the seeming contradiction of the existence of a (1-1) correspondence between the points of a line to which we should wish to assign the dimension one and the points of a plane which should be of dimension two. This line of development runs from Poincaré through Brouwer to Menger and Urysohn. The whole theory is based upon the three axioms fundamental in the Fréchet theory plus two additional axioms, that of normality and the second countability axiom, that is, a space in which distance can be defined. The author discusses the theorem of Lebesgue concerning intersection of coverings which can be used to give a definition of dimension independent of recurrence and which is usually used to show that the Menger-Urysohn definition gives the dimension n to sets to which we would intuitively assign the same dimension. A brief discussion is also given of the combinatorial point of view with respect to dimension which has been so ably developed by the powerful methods of Alexandroff.

(3) The definitions based on the notion of measure. The starting point here is the linear measure of Carathéodory, that is, measure of order unity. There is then given an outline of the generalizations due mainly to Hausdorff and the relation of this to the work on potential theory done mainly about the middle of the last decade. There is also a brief discussion of the work on transfinite diameter in which the names of Pólya, Szegö, and Fekete play the prominent role.

This book furnishes an excellent sketch of the various points of view and serves as an introduction to a more detailed study for which there are available Menger's book on dimension, Fréchet's *Espaces Abstraits*, and a host of original articles.

J. R. KLINE

Versicherungsmathematische Aufgabensammlung. Vol. 1. *Beiträge und Deckungsrücklagen in der Lebensversicherung.* By C. Boehm and E. Rose. 75 pp. Vol. 2. *Umwandlung von Lebensversicherungen.* By C. Boehm and P. Lorenz. 52 pp. Leipzig and Berlin, Teubner, 1937.

These two pamphlets contain a collection of problems with their solutions illustrative of the more simple types of calculations of an actuarial nature which are required in actual life insurance practice. Although the authors very properly point out in the two prefaces that no book can be a substitute for actual experience, these pamphlets are written from a more practical standpoint than most textbooks and provide an interesting and valuable insight into the practice of German life insurance companies. The first volume deals with the calculation of net single premiums, net annual premiums, gross premiums, net reserves, gross reserves, and special plans of insurance, in that order. The examples seem, on the whole, well chosen, and cover the ground well. The argument against the use of ultraconservative interest and mortality bases on page 12 seems to the reviewer rather naïve and unconvincing. On page 27 in a problem to determine what per cent of the gross premium the various elements of expense constitute, the collection cost is expressed as a percentage of the entire gross premium, while in the case of the clerical and administrative expense and the prorated initial expense the denominator used is the gross premium less the collection expense. This may be the German custom but would seem to call for a word of explanation. The statement on page 51 that the prospective formula is the simplest for calculating net reserves is true, in general, but it is odd that it should occur in the discussion of a special varying insurance for which the retrospective formula would have been easier.

The second volume gives the impression that the German treatment of the difficult problem of policy changes is characterized by the same balancing of theoretical accuracy against practical expediency which is typical of our own approach to the subject. The per mille symbol referred to by Professor Dodd in the September 1938 Bulletin in reviewing a similar book is used frequently, and by its similarity to the per cent sign may confuse the reader to whom it is not familiar. The explanations are clear, and, generally speaking, these books have accomplished their purpose in an admirable fashion.

T. N. E. GREVILLE

Stellar Dynamics. By W. M. Smart. Cambridge, University Press; New York, Macmillan, 1938. 8+434 pp.

This book is an effort to present, in considerable detail, the development of stellar dynamics from a mathematical treatment of the results of observations. The general theory of the correction of observed facts is treated and results of this treatment are applied to the correction of parallaxes, absolute magnitudes, and transverse velocities.