first order. Since that time many brilliant mathematicians have contributed to the development of the calculus of variations and some have noticed the connection announced by Jacobi. But it has remained for Professor Carathéodory to give a systematic account of the subject in the book under review.

The book is divided into two parts. The first part (163 pp.) is devoted to partial differential equations of the first order and is an elegant presentation of the theory which is necessary to bring out the relationships with the calculus of variations, which forms the second part of the book. The author's treatment of the simplest problem of the calculus of variations leads directly to the Hamilton-Jacobi equations of mechanics and the foundations of the subject are fully discussed.

The author has not attempted to give a complete treatment of the later developments but has intended to carry them far enough to connect with existing literature. For this purpose the bibliography, containing 190 references to books and articles, is followed by a very helpful section giving advice on the use of the literature.

To quote from the preface: "The purpose which I have pursued in this book will have been accomplished if the student of this field of mathematics may become convinced that there are today three principal aspects of the calculus of variations: first, the calculus of variations of Lagrange, which today forms a part of the tensor calculus; second, the theory of Tonelli, in which the finer aspects of the minimum problem are based on point set theory; then the point of view presented in this book, which is oriented to the theory of differential equations, differential geometry, and physical applications, which was first made prominent by Euler. I hope also to have shown that the Weierstrass theory belongs to this last aspect."

Professor Carathéodory is not only a master of his subject but also a master of presentation. This clearly written fundamental work will prove indispensable for all students of the subject.

W. R. LONGLEY

Introduction to the Theory of Linear Differential Equations. By E. G. C. Poole. London, Oxford University Press, 1936. 200 pp.

This book deals with ordinary linear differential equations and is based on lectures delivered by the author to senior undergraduates at Oxford. Natural prerequisites are an elementary course in differential equations and considerable familiarity with the theory of matrices. The selection of material is such as to form an excellent introduction to this vast and important field of mathematics.

The first half of the book deals with properties common to wide classes of equations. The first chapter, on existence theorems, presents the fundamental theorems for given initial conditions. In the second chapter the solutions of equations with constant coefficients, subject to initial conditions of the Cauchy type, are obtained by the methods of the Heaviside operational calculus. The third chapter is concerned with some formal investigations involving linear operators, adjoint equations, and simultaneous equations with variable coefficients. The next two chapters deal with equations having uniform analytic coefficients, attention being focused on regular singularities. In the second half of the book the author chooses for detailed study the hypergeometric equation, Laplace's linear equation, and the equations of Lamé and Mathieu.

The style of the book is somewhat terse and, while usually clear, is not always easy reading for one approaching the subject for the first time. This difficulty is largely overbalanced by abundant references to the literature so that an industrious reader will be able to learn very much with this volume as a basis for his study. The details given in the text are considerably extended by the collection of examples at the end of each chapter. A large number of these examples contain citations to their origin in the literature.

W. R. LONGLEY

La Théorie du Potentiel et ses Applications aux Problèmes Fondamentaux de la Physique Mathématique. By N. M. Gunther. Paris, Gauthier-Villars, 1934. 303 pp.

This volume is one of the collection of monographs on the theory of functions published under the direction of Émile Borel and contains a carefully written and rigorous treatment of the material usually covered in an introductory course on the theory of the newtonian potential function.

The first chapter contains general definitions and theorems concerning functions of the type to be encountered later. The characteristic properties of the newtonian potential of a three-dimensional distribution of attracting matter and of a simple and of a double surface distribution are treated in the second chapter. The remaining three chapters are concerned with the standard problems associated with the names of Neumann, Robin, Dirichlet, and Green.

The book is entirely self-contained in that it contains only one reference to the literature and this occurs in the last paragraph of an appendix.

W. R. LONGLEY

Le Problème de la Dérivée Oblique en Théorie du Potentiel. By G. Bouligand, G. Giraud, and P. Delens. Paris, Hermann, 1935. 78 pp.

This is No. 219 of the series *Actualités Scientifiques et Industrielles* and No. 6 of the subseries devoted to geometry and edited by E. Cartan. The problem treated is the extension of the Neumann problem in potential theory where the normal derivative is replaced by a directional derivative whose direction is prescribed over the bounding surface. When the direction is never tangent to the bounding surface and when, in addition, the direction cosines fixing the direction and the values assigned to the directional derivative satisfy certain regularity conditions (Hölder and continuity conditions) the problem is termed regular. The first part of the present work, written by Bouligand, is introductory and shows the essential character of the criterion of regularity (when the direction of differentiation can become tangent to the bounding surface the uniqueness theorem which holds in the regular problem breaks down). The second part, written by Giraud, gives the solution of the regular problem (under a stated condition of compatibility on the assigned values of the directional derivative). The third part, by Delens, discusses the connection of the theory of congruences of curves with the problem. This arises through a study of harmonic functions of the form $\phi(\alpha, \beta)$, where α, β are functions of position.

F. D. MURNAGHAN