This suggestion from physics becomes more interesting when we note in pure mathematics that the rigorous formulation of the calculus necessitated assumptions by Dedekind and Cantor which lead to unresolved contradictions. Similar considerations have suggested to this reviewer that the theoretical difficulties at the basis of physics and mathematics may have much more in common than has been realized, and that a clue to their resolution may be found in an alteration in the more general philosophical assumptions common to the two sciences.

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L'Arithmétique dans les Algèbres de Matrices. By Claude Chevalley. (Actualités Scientifiques et Industrielles, No. 323.) Paris, Hermann, 1936. 33 pp.

This book is one of the collection called *Exposés Mathématiques*, published in memory of the late Jacques Herbrand.

This investigation represents another step in the simplification by abstraction of the number theory of linear algebras, a theory which seemed so impossibly complicated when it was first attacked but a few years ago. Ideal theory has grown in importance until, as in the present paper, it constitutes the whole of arithmetic.

Let $\mathfrak{S}$ be a ring with unit element in which every regular element (that is, not a divisor of zero) has an inverse. Then $\mathfrak{S}$ has a regular arithmetic when there is defined a system of modules $\mathfrak{A}, \mathfrak{B}, \cdots, \mathfrak{O}, \cdots$, called ideals such that:

I. The ideals form a groupoid under modular multiplication. The left (right) order of an ideal $\mathfrak{A}$ is the totality $\mathfrak{O}(\mathfrak{O}')$ of elements $\lambda(\lambda')$ such that $\lambda\mathfrak{A}\subset\mathfrak{A}(\mathfrak{A}\lambda'\subset\mathfrak{A})$. The inverse $\mathfrak{A}^{-1}$ of $\mathfrak{A}$ is the set of elements μ such that $\mu\mathfrak{A}\subset\mathfrak{O}'$, $\mathfrak{A}\mu\subset\mathfrak{O}$.

II. If $\mathfrak{A}$ is an ideal, every element of $\mathfrak{S}$ is a product of an element of $\mathfrak{A}$ by the inverse of a regular element of $\mathfrak{A}$.

III. If $\mathfrak{O}$ is a unit of the groupoid, the ideals which have $\mathfrak{O}$ for their left order are the finite left $\mathfrak{O}$-modules which contain regular elements.

IV. In every class of left or right ideals there is an integral ideal prime to any given integral ideal.

The principal result obtained is that if $\mathfrak{S}$ is a total matric algebra over a (not necessarily commutative) field k in which a regular arithmetic is defined, one can define a regular arithmetic in $\mathfrak{S}$. It is known that such a regular arithmetic can be defined in every simple algebra whose centrum is an algebraic field. We see the importance of this result if we recall Wedderburn's theorem that every simple algebra is a total matric algebra over a division algebra.

The above theorem leads to further results in the theory of ideal classes in $\mathfrak{S}$.

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Variationsrechnung und partielle Differentialgleichungen erster Ordnung. By C. Carathéodory. Teubner, Leipzig and Berlin, 1935. 11+407 pp.

A century ago Jacobi, influenced by Hamilton's work on geometrical optics, discovered that there exists a direct connection between the theory of the calculus of variations and the theory of partial differential equations of the