surability is infinitely improbable and that therefore $N_i(t)$ is not periodic in practice implies the existence of a criterion of probability for the $n/2$ periods, although such a criterion would necessarily be less elementary than the criterion of periodicity of $N_i(t)$—which is presumably estimated directly from the biological system.*

This criticism is on something irrelevant to the real purpose of the book, as stated by the author in the sentence quoted above. The mathematical content of the book, a detailed analysis of certain differential and integro-differential equations, should be valuable both to the mathematician and to the statistician interested in the qualitative and quantitative study of the development of a biological system.

J. L. DOOB

Interpolatory Function Theory. By J. M. Whittaker. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 33.) Cambridge University Press. New York, Macmillan, 1935. vii+107 pp.

This interesting tract by the son of E. T. Whittaker is devoted to the following problem. Let $\Pi_i f(0)$ denote the differential operator

$$\Pi_i f(0) = \sum_{j=0}^{\infty} \frac{\pi_{ij}}{j!} f^{(j)}(0).$$

What analytic functions $f(z)$ are uniquely determined by the values of $\Pi_i f(0)$, $i=0, 1, 2, 3, \cdots$? This problem contains a large number of important special cases such as expansion in Taylor's series, interpolation at positive integers or at lattice points, and Bernoullian series. The fundamental concept is the notion of basic sets. A set of polynomials

$$p_i(z) = \sum_{j=0}^{\infty} p_{ij} z^j$$

is said to be *basic* if every polynomial is a linear combination of a finite number of polynomials from the set. A necessary and sufficient condition that the set be basic is that the matrix $P = \|p_{ij}\|$ have a row-finite inverse. A set of operators is basic if the set of associated functions

$$p_i(z) = \sum_{j=0}^{\infty} \pi_{ji} z^j$$

is a basic set of polynomials, that is, if the transpose of the matrix $\Pi = \|\pi_{ij}\|$ is row-finite and has a row-finite inverse. The series

$$\sum_{i=0}^{\infty} \Pi_i f(0) p_i(z)$$

* There seems to be a general principle in many investigations that the constants which occur in it have a continuous probability distribution, so that it is "infinitely improbable" that these constants have any preassigned set of values. As in the case just discussed, this unjustified assumption may be almost precisely what was to be proved.

associated with a basic set of operators $\Pi_i f(0)$ is called the *basic series*. It represents $f(z)$ if $f(z)$ is a polynomial or an entire function of sufficiently low order, but may in exceptional cases represent a much wider class. Thus, if Π is the unit matrix, the basic series is simply that of Maclaurin which represents all functions regular at $z=0$.

So much for the general problem. The tract is divided into six chapters with the following headings: 1. *Series of polynomials*; 2. *The sum of a function*; 3. *Properties of successive derivatives*; 4. *Interpolation at the integers*; 5. *Interpolation at the lattice points*; 6. *Asymptotic periods.*

The first chapter deals with basic sets and the convergence of basic series. In the second chapter the problem is that of finding a function $g(z)$ such that $g(z+1)-g(z)=f(z)$, where $f(z)$ is a given entire or meromorphic function and a solution of the same nature is desired. The underlying operators are associated with the Bernoulli polynomials which form a basic set. Chapter 3 contains three somewhat loosely connected topics: (i) Pólya's theorem on the infinitary behavior of the zeros of the derivatives of a meromorphic function; (ii) Gontcharoff's theorem on the convergence of the basic series associated with the operators $f^{(n)}(a_n)$; (iii) the author's investigation of Lidstone's series, that is, the basic series associated with the operators $f^{(2n)}(0)$, $f^{(2n)}(1)$. The first half of Chapter 4 deals with the binomial or Gregory-Newton series, that is, the basic series for the operators $\Delta^n f(0)$, and includes some theorems on relations between singularities and coefficients of power series. The second half is mainly concerned with the cardinal series

$$\frac{\sin \pi z}{\pi}\left[\frac{f(0)}{z}+\sum_{n=1}^{\infty}(-1)^n\left\{\frac{f(n)}{z-n}+\frac{f(-n)}{z+n}\right\}\right],$$

the author's first work in mathematics. In Chapter 5 the two-dimensional cardinal series is introduced where $\sin \pi z$ is replaced by the Weierstrass sigma function. The main part of the chapter is concerned with "flat" regions of entire and meromorphic functions. Chapter 6 is rather loosely connected with the rest, but its results are in a sense converses of those of Chapter 2. A quantity $\beta\neq 0$ is an asymptotic period of an entire or meromorphic function $f(z)$ if $f(z+\beta)-f(z)$ is of lower order than $f(z)$. The author has determined the structure of the set of asymptotic periods which reveals a striking difference between the entire and the meromorphic cases.

It has already been observed that the tract is largely based upon the author's own research. The result is an interesting but also somewhat one-sided selection of material. Even if the author had to omit much interesting work owing to lack of space (see his remarks in the preface), he could perhaps have included related papers in the bibliography. A reference to Gelfond's work on the transcendental character of e^{π} could have been included in the chapter on interpolation at the lattice points. The experts will undoubtedly notice other and perhaps more important omissions. But within the somewhat limited field the author has produced an interesting, readable, and highly stimulating booklet. For this service we render thanks.

EINAR HILLE