

are treated. Then follows a discussion of ruled surfaces, rational surfaces, Veronese's quartic and its generation in  $S_n$ , and finally surfaces whose plane sections have a given genus.

The next chapter deals with manifolds of dimension greater than two, the discussion being similar to that for surfaces. Hypersurfaces are studied in detail, including their characteristics, systems, postulation, intersections with manifolds. Seven pages are given to cubic hypersurfaces. Manifolds generated by a single infinity of lines and other special manifolds are treated in the latter part of the chapter.

In Chapter 8 there are developed geometries in which any linear  $S_k$  is the element. Chapter 9 contains a very brief discussion of the principle of correspondence in  $S_n$ .

In Chapter 10, the concept of hyperalgebraic geometry is presented. The manifolds of hyperalgebraic geometry have equations in the coordinates of conjugate imaginary elements. The study of linear transformations connecting hyperalgebraic manifolds leads to antiprojectivities and their special cases. An antiprojectivity is a linear transformation in which the anharmonic ratios of four elements and their four corresponding elements have conjugate imaginary values. These ideas were developed by Segre in 1890–91. He seems not to have known that they had been treated previously in a thesis by a Danish geometer, C. S. Juel, under the name of "symmetralities." This thesis was published in Copenhagen in 1885 and later in the *Acta Mathematica* (1890). No reference to Juel is given in this monograph.

There is no separate index since the volume is indexed as a whole. The brief but pithy contents, however, present an excellent outline of the subject matter.

There is evidence that Segre did not particularly enjoy writing this monograph—in the decade or more preceding, he had become interested in other phases of mathematics. He did the work, however, with an exactitude, finesse, and comprehension that have rarely been equalled. One who studies it will readily agree with Loria\* that the difficult task of writing a digest of the geometry of  $n$  dimensions was performed with such great care and insight that this article deserves to serve as a model for future similar works.

T. R. HOLLCROFT

*Algebraische Liniengeometrie.* By Konrad Zindler. Band III 2, Heft 8. December, 1922. Pages 973–1228.

The author of this report had three decided advantages: the subject was comparatively new and sharply defined, he had contributed a considerable part of the literature himself, and was still in the prime of productivity when the report was written. The results of these conditions are everywhere apparent in a comprehensive and well-rounded product.

It is curious that a subject that was practically unknown three quarters of a century ago is now an indispensable tool in algebraic and projective geometry besides furnishing a vast laboratory in the foundations of mathematics.

The first section, about 100 pages, is devoted to the elementary concepts,

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\* G. Loria, loc. cit.

systems of coordinates, methods of mapping, linear transformations, and an exhaustive discussion of the linear complex, pencils of complexes and of reguli.

The remaining sections are devoted respectively to algebraic complexes, congruences, and ruled surfaces. The quadratic complex and systems of non-singular quadratic complexes are discussed in such detail as to supply a helpful addition to existing literature in the way of a hand-book. Congruences are nearly as fully treated, especially those contained in a quadratic complex. Applications to the Kummer surface are even better coordinated than in the existing textbooks.

The summary of algebraic ruled surfaces, comprising over a hundred citations, is exceptionally well done. Since it was written, the only important additions are the book of Edge and the essay of Wiman in the *Acta Mathematica*.

The report was published in the most trying post-war period. This explains the omission of three chapters: differential line geometry, geometry of the sphere, and the use of other space elements, each of which could contribute materially to the usefulness of the report.

VIRGIL SNYDER

*Algebraische Raumkurven und abwickelbare Flächen.* By K. Rohn and L. Berzolari. Band III 2 B, Heft 9. July, 1926. Pages 1229-1436.

This volume covers approximately a century of research on space curves and developable surfaces. The author is well fitted for the task, having been familiar with and a contributor to the field for more than a third of that time. There is an abundance of material and its arrangement shows very careful planning, for it reads more like a textbook than the source book it really is.

In some places the material is covered in full, in others it is barely sketched. However, in either case the reader is liberally supplied with references. The references given are frequently really footnotes as they contain additional and valuable information upon the subject.

The first two-thirds of the book deals principally with general properties of space curves and their developable surfaces. It begins with some definitions and representations of a space curve, and includes many of the fundamental properties resulting from these definitions. Singular points are introduced early, followed by the Cayley and other formulas. The geometry on a curve and the classification of curves are discussed at some length.

The latter third of the book deals primarily with particular curves and surfaces. Rational curves of order four, five, six, and seven, and irrational curves of order four, five, and six are discussed. Some other special curves as well as special systems of curves are also discussed.

AMOS BLACK

*Spezielle Algebraische Flächen.* By W. Fr. Meyer. a. *Flächen dritter Ordnung.* Band III 2, Heft 10. 1928. Pages 1437-1531; b. *Flächen vierter und höherer Ordnung,* Heft 11. 1931. Pages 1533-1779.

The discussion of cubic surfaces is divided into two parts. The first includes the older historic treatment, mapping, and the development of properties; the second part reviews the prize papers of Cremona and Sturm, and outlines