through the center of gravity and the vertices and so is a sphere.

The statement of our main theorem can be given in more general form but our statement is chosen on account of its intuitive simplicity. The set R we may take as merely closed and bounded; S may be the frontier of a bounded domain, D, which contains R. Then the conclusion remains the same as we have stated it in the simpler case.

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A DECOMPOSITION THEOREM FOR CLOSED SETS*

BY G. T. WHYBURN

Let P be any local† topological property of a closed set such that if K is any compact closed set lying in a metric space, then the set of all non-P-points of K is either vacuous or such that its closure is of dimension >0. The following are examples of such properties: (i) local connectivity, (ii) regularity (Menger-Urysohn sense), (iii) rationality, (iv) being of dimension $<n$, (v) belonging to no continuum of convergence, (vi) belonging to no continuum of condensation. In fact, it will be noted that in each of these cases, every non-P-point of a compact set K lies in a non-degenerate continuum of non-P-points of K. We proceed to prove the following theorem.

THEOREM. *If N denotes the set of all non-P-points of a compact closed set K in a metric space and if K is decomposed upper semi-continuously‡ into the components of $\overline{N}$ and the points of $K-\overline{N}$, then every point of the hyperspace H is a P-point of H.*

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† For the purposes of the present paper we shall understand by a local property of a set K a point property P such that if some neighborhood of a point x in K has property P at x, then K has property P at x; and conversely, if K has property P at x, then any neighborhood of x in K also has property P at x. A point x of K will be called a P-point or a non-P-point of K according as K does or does not have property P at x.

‡ For the notions relating to upper semi-continuous decompositions and for a proof that our particular decomposition is upper semi-continuous, the reader is referred to R. L. Moore, *Foundations of Point Set Theory*, American Mathematical Society, Colloquium Publications, 1932, Chapter 5.

PROOF. Let T denote the continuous transformation of K into H associated with our decomposition,* that is, $T(K)=H$ and for each $x\epsilon H$, $T^{-1}(x)$ is either a component of $\overline{N}$ or a point of $K-\overline{N}$. Then clearly $T(\overline{N})$ is a set of dimension 0. Thus if we suppose, contrary to our theorem, that H has at least one non-P-point, it follows by the condition on P that the set of all non-P-points of H can not be contained in $T(\overline{N})$. Hence there exists at least one point x in $K-\overline{N}$ such that $T(x)=y$ is a non-P-point of H. Let U be a neighborhood of x in K such that $U\cdot\overline{N}=0$. Then on U, T is a homeomorphism and $T(U)$ is a neighborhood of y in H. Since P is a local property and x is a P-point of K, x is also a P-point of U. Thus y is a P-point of $T(U)$ as U and $T(U)$ are homeomorphic; but then y must be a P-point also of H, because $T(U)$ is a neighborhood of y in H. Thus our supposition that not every point of H is a P-point leads to a contradiction and our theorem is proved.

COROLLARY. *If K is a compact metric continuum, then, choosing the properties* (i)–(vi) *above in turn, we find that the hyperspace of the decomposition is, respectively,* (i) *a locally connected continuum,* (ii) *a regular curve,* (iii) *a rational curve,* (iv) *a continuum of dimension $<n$,* (v) *an hereditarily locally connected continuum, and* (vi) *a continuum having no continuum of condensation.*

Part (i) of this corollary will be recognized at once as the well known result of R. L. Moore† to the effect that any continuum is a continuous curve with respect to its prime parts, for clearly the decomposition in this case is identically the decomposition of K into its prime parts.

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* See Alexandroff, Mathematische Annalen, vol. 96 (1926), p. 555; and Kuratowski, Fundamenta Mathematicae, vol. 11 (1928), p. 169.

† See Mathematische Zeitschrift, vol. 22 (1925), p. 308. For the notion of a prime part of a continuum together with a similar result for irreducible continua, see Hahn, Sitzungsberichte der Akademie der Wissenschaften in Wien, Mathematisch-Naturwissenschaftliche Klasse, vol. 130 (1921), pp. 217–250.