COHEN-NAGEL ON LOGIC

An Introduction to Logic and Scientific Method. By Morris R. Cohen and Ernest Nagel. New York, Harcourt Brace, 1934. xii+467 pp.

The plan and purpose are indicated by the following extracts from the preface: "The present text aims to combine sound logical doctrine with sound pedagogy and to provide illustrative material suggestive of the role of logic in every department of thought." . . . "It is especially the hope of the authors that general readers as well as students of the natural and social sciences will find this book helpful towards an understanding of scientific method."

The work consists of two Books. Book I, *Formal Logic*, is a complete and lucid exposition of the formal logic of Aristotle, followed by a selection of material from modern authors, for example, de Morgan, Boole, Peirce, Whitehead and Russell, blended into a consistent whole. Book II, *Applied Logic and Scientific Method*, is an exposition of the logic of probable inference. The authors first clear the ground by pointing out errors in former treatises on inductive logic, then lay the foundations for an improved exposition by means of chapters on *Probability and Induction*, *Measurement*, and *Statistical Methods*. They show that the structure erected in Book I can be placed on these foundations, and illustrate the scientific method thus constituted by applying it to various fields of knowledge. Chapter 1 forms an introduction to the whole. Book II is followed by an Appendix on demonstrations, Exercises, and the Index.

At least a fourth of the book treats mathematical topics. Chapter 6 contains the formal properties of relations, the calculus of classes, and the calculus of propositions. Chapter 7 presents Veblen and Young's postulates for projective geometry, isomorphism, equivalence of axiom sets, independence and consistency of axioms, and mathematical induction. Chapter 8 contains a section on probability. Chapter 14 discusses sampling. Chapter 15 describes the nature of counting, and the measurement of intensive and extensive qualities. Chapter 18 is devoted to statistical averages, measures of dispersion and correlation, and dangers and fallacies in the use of statistics.

The work is pleasing and satisfactory. The chapters on *Aristotelian and Mathematical Logic*, *Methods of Experimental Inquiry*, *Statistical Methods*, and *Probable Inference in History*, are especially good. The method of presentation is excellent. Every chapter is introduced by an explanatory section. Every general statement, every passage which might otherwise be obscure, is explained by instances. Interest is stimulated by the occasional introduction of unconventional expressions, humorous comments, and fanciful examples. The style is flowing and rhythmical, but discursive in places. Occasionally, the combination of long sentences and paucity of punctuation is disconcerting (see p. 233, lines 34–37; p. 356, lines 23–27). The vocabulary is extensive, and includes words, such as equiprobable, musicology, which are not in the dictionaries. The book is well printed in all respects.

The authors' presentation of the scientific method differs in an important respect from that actually used in the exact sciences: scientists do not, except to a very limited extent, employ traditional logic in drawing their inferences.

Various writers (for example, Mariotte, Oeuvres, pp. 609–701, Leyden, 1717) have attempted to describe scientific logic, but scientists do not adhere closely to any one model. Reasons for this non-employment of traditional logic are: (1) It is difficult to acquire a clear understanding of it, because the texts do not define fundamental logical terms. (2) The types of propositions used in the exact sciences differ from those discussed in traditional logic. Thus the exact sciences require only one type of categorical proposition (All A's are B's). On the other hand, they frequently use implications of the type "P implies Q", where P and Q are classes of propositions; such implications are not separately discussed in formal logic, though they have distinctive properties.

However, the present work makes some approach toward the scientific point of view. Logical terms are usually defined; true, false, hypothetical proposition, and one or two others, being exceptions. Though there are few changes in the traditional doctrine, there are additions which make it more serviceable. The introduction of the chapters on mathematical logic is important.

Other grounds for criticism are few, and may be briefly summarized. The statement (p. v) "We do not believe that there is any non-Aristotelian logic · · · in which the contraries of the Aristotelian principles of contradiction and excluded middle are assumed to be true, and valid inferences drawn from them," is disparaging to the work of Brouwer. If "implies" is made to correspond to "contains," the analogy (p. 126) between classes and propositions becomes somewhat closer. The distinction between intensive and extensive qualities is not so sharp as the account in Chapter 15 suggests; authority can be cited (see Newton, *Principia*, Def. 1) for the opinion that density is an extensive quality. On page 293 we read that "The distinctive feature of (sampling) consists in concluding that the proportion of characters found in the sample is the same as the proportion found in the entire collection." It is, of course, the *limit* of the proportion of characters, as the number of samples increases indefinitely, which is concluded to be the same as in the collection. On page 374 it is suggested that "surd" means "absurd"; in fact, the word is derived from *surdus*, hence its meaning appears to be "not clearly defined."

The following are corrections of minor errors: page 17, line 20: for "real" read "algebraic"; page 66, lines 37–38: second is true . . . first; page 81, line 14: read "term"; page 97, line 31: for "1" read "2 and 3"; l. 35, for "2 and 3" read "1"; page 126, line 28: the last half of the line should not be italicized; page 194, line 36: for "including" read "up to and including"; page 207, lines 7, 8: the phases of Venus were first observed in the seventeenth century; page 210, line 10: read "rejected"; page 266, line 17: read "n H's"; page 312, lines 21, 22: read "If D is the quartile deviation, one half of the items will lie between Q_2-D and Q_2+D"; page 315, line 2 below the table: *dele* numerical.

Symbolic Logic, by Lewis Carroll (C. L. Dodgson), is frequently quoted. That book and the two following books should be included in the bibliography:

The Art of Scientific Discovery, G. Gore (London, 1878),

The Grammar of Science, Karl Pearson.

This work can be recommended to all mathematicians who are interested in logic. It presents a clear exposition of the fundamentals of that subject.

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