ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

219. Dr. W. H. Ingram: *A necessary condition for the operability of systems of synchronous alternators.*

In the case of three synchronous smooth-pole slip-ring machines connected together at a point, the *Lagrangean* multiplier is given by $e_0=\sum(z_0|z_k)e_k^*\cdot\psi_k-\rho_k)$, and the condition of operability is that real ψ's exist which satisfy the generalization of Hopkinson's equation $p_i=\sum e_i^*\cdot e_j^* z_{ij}^{-1}(k_{ij}-\delta_{ij}\lambda_i)$, where δ_{ij} is Kronecker's symbol, $k_{ij}=\sin(\psi_i+\alpha_i)\cos(\psi_j-\rho_j)$, $\lambda_i=(z_i|z_0)\sin\alpha_i$. A limit of synchronous operability is reached when any one of the derivatives $\partial p_i/\partial\psi_j$ vanishes. The ψ's are given explicitly by elliptic and, in the case of salient-pole machines, hyperelliptic functions; and more practically by a system of pendulums. The method of solution can be extended to unbalanced 3-phase systems without difficulty when the field currents are constant. (Received May 17, 1:34.)

220. Professor W. M. Whyburn: *Study of a series of the binomial type.*

The series $F[v, f(x)]=\sum_{i=0}^{\infty}\binom{v}{i}u_i$, where $u_i=\sum_{j=0}^{i}\binom{i}{i-j}f^{(j)}(x)(-1)^{i-j}$, v is a real number, and $\binom{v}{i}$ is a generalization of the binomial coefficient, is studied. Connection is made with previous investigations of derivatives of non-integral orders and with work on infinite systems of differential equations. (Received May 21, 1934.)

221. Professor D. N. Lehmer: *On the enumeration of magic cubes.*

The author devises a method of "normalizing" magic cubes so that the enumeration of them is made proof against repetition or omission. The enumeration has been carried out for cubes of order 3 with the result that there are exactly 4 normalized cubes each representing a group of 1,296 different cubes. The total number of magic cubes of order 3 is therefore 5,184 (Received May 19, 1934.)

222. Professor J. V. Uspensky: *On an expansion of the remainder in the Gaussian quadrature formula.*

The remainder in the Gaussian quadrature formula with n ordinates and corresponding to the interval of integration 0, 1 can be expanded into series of the type

$$C_0\{f^{(2n-1)}_{(1)} - f^{(2n-1)}_{(0)}\} - C_1\{f^{(2n+1)}_{(1)} - f^{(2n+1)}_{(0)}\} + C_2\{f^{(2n+3)}_{(1)} - f^{(2n+3)}_{(0)}\} - \cdots,$$

where $C_0, C_1, C_2, \cdots$ are positive rational numbers. This expansion is analogous to one figuring in the well known Euler-Maclaurin formula and possesses the same properties. The method by which the above expansion is obtained leads, incidentally, to a new and very simple proof of some important inequalities due to Tchebycheff. (Received May 25, 1934.)

223. Dr. C. B. Morrey, Jr.: *New differentiability properties of the solution of a class of minimum problems.* Preliminary report.

Let Γ be a regular curve in space which has a convex projection C (bounding the region R) on the (x, y) plane and possesses a continuously turning osculating plane which never makes an angle $> \tan^{-1}\Delta$ with that plane. Let $f(p, q)$ be defined, of class C'', and satisfy $f_{pp}f_{qq}-f_{pq}^2>0$, $f_{pp}>0$, for all real (p, q). Radó and Haar have shown that there exists a unique function $z_\Delta(x, y)$ which (1) satisfies a Lipschitz condition with constant Δ on $R+C$, (2) is such that $z=z_\Delta(x, y)$ is bounded by Γ, and (3) is such that

$$\int\int f\left(\frac{\partial z_\Delta}{\partial x}, \frac{\partial z_\Delta}{\partial y}\right) dxdy$$

is less than the value of this integral formed for any other function $z(x, y)$ which satisfies a uniform Lipschitz condition on R and coincides with z_Δ on the boundary. The present paper shows that the first partial derivatives of z_Δ are absolutely continuous in the sense of Tonelli on every closed subregion interior to R and that *their* first partial derivatives (the second partials of z_Δ) are summable with their squares on each such subregion. The function z_Δ is then seen to satisfy the Euler equation almost everywhere. (Received May 26, 1934.)

224. Professor Morgan Ward: *The interpretation of a many-valued logic as an isomorphism between rings.*

The rings in question are the Boolean algebras in which the elements 0 and 1 need not appear, studied by F. Klein. The logics of Tarski and Lukasiewicz and C. I. Lewis appear as special instances of a general correspondence between such systems. (Received May 28, 1934.)

225. Professor Raymond Garver: *A note on group postulates.*

This paper shows that the set of four group postulates which E. H. Moore devnoted by W_1' in volume 3 of the Transactions of this Society is not independent. (Received May 29, 1934.)

226. Professor A. R. Williams: *The quadrics of a web that have a line of vertices.*

In a general web of quadrics in S_3 are 10 pairs of planes, that is, 10 quadrics, each of which has a line of vertices. An algebraic method of finding this number is given and extended to a web of quadrics in S_n. (Received May 29, 1934.)

227. Professor B. A. Bernstein: *On finite Boolean algebras.*

The author brings out certain facts concerning the "irreducible" elements of a finite Boolean algebra, gives a new proof of the theorem concerning the number of elements in such an algebra, and answers some questions concerning finite sub-algebras in a given Boolean algebra. (Received May 31, 1934.)

228. Professor A. D. Michal and Mr. V. Elconin: *An existence theorem for total differential equations in abstract vector spaces.*

In this paper the authors give an existence theorem for completely integrable Pfaffian equations $\delta\phi(x)=F(x, \phi(x), \delta x)$ in Banach spaces. Certain hypotheses on F are introduced to insure the existence of first and second Fréchet differentials of the solution function $\phi(x)$. Implicit function theorems in Banach spaces are then introduced as applications of the above existence theorem. Some attention is given to the consideration of instances. (Received May 31, 1934.)

229. Professor H. F. Blichfeldt: *Minimum values of quadratic forms.*

Consider a positive definite quadratic form reduced by the method of Korkine and Zolotareff, its outer coefficients being $A_1, A_2, \cdots, A_n$. From certain relations among these, virtually equivalent to $2A_5 \geqq A_1$, in addition to those established by Korkine and Zolotareff, can be deduced the values of the minima of the quadratic forms of given determinant $D=A_1A_2\cdots A_n$ in $n<9$ variables when these are integers not all zero. The results will appear shortly in the Mathematische Zeitschrift. An extension is now made to forms in nine variables, establishing $A_1^9 \leqq 2^{10}D$. When $n>10$, the relation $2A_5 \geqq A_1$ (or its equivalent) ceases to give as good results as the general limit given by the author (see Mathematische Annalen, 1929, pp. 605–8). (Received May 31, 1934.)

230. Dr. D. H. Lehmer: *On the inversion of infinite series.*

In a previous paper (Transactions of this Society, vol. 33, pp. 945–957) we developed a general calculus of numerical functions associated with a function $\psi(x, y)$ satisfying certain postulates. In this paper we use this calculus to obtain general inversion formulas for infinite series whose nth terms are of the form $a_nF(V_n(z))$ where the a's and F are arbitrary while the V's are functions of a complex variable and form a closed set with respect to substitution as follows: $V_i(V_j(z))=V_k(z)$ where $k=\psi(i, j)$. The case $\psi(i, j)=ij$ includes most series developments in common use. A number of other examples are given to illustrate the general theory. (Received June 1, 1934.)

231. Mr. Irving Kittell: *The coloring of spherical maps.*

In this paper it is shown that a minimum uncolorable spherical map cannot have n-fold symmetry with respect to a point if $n>3$, or if $n\leqq 3$ unless a point of symmetry is on a 0-cell or a 1-cell. The work of A. Errera on transpositions of chains in a partially colored map is extended by giving a group of transpositions which could be performed upon a minimum uncolorable map. Also a symbolic notation is presented which will greatly facilitate the coloring of any colorable spherical map. (Received May 22, 1934.)

232. Mr. P. B. Jones: *Concerning certain locally peripherally connected spaces.*

A space is said to be locally peripherally connected provided that if P is a point of a region R, there exists in R a domain containing P and whose boundary is connected. Suppose that M is a locally peripherally connected, completely separable space satisfying Axioms 0, 1, 2, 3, and 4 of R. L. Moore's *Foundations of Point Set Theory*. In this paper the author shows that M is homeomorphic with a subset of a completely separable space S having the following properties: (1) S satisfies Moore's Axioms 0, 1, 2, 3, and 4; (2) if P is a point of a region R of S, there exists a simple domain D of S lying in R and containing P. By a theorem of J. H. Roberts (this Bulletin, vol. 39 (1933), p. 620), S is homeomorphic with a subset of a sphere. Hence, M is homeomorphic with a subset of a sphere. If M is not compact, M is homeomorphic with a subset of a plane. (Received May 23, 1934.)

233. Professor Morgan Ward: *Note on the period of a mark in a finite field.*

A partial determination of the period of a mark in a finite field is effected. This paper will appear shortly in this Bulletin. (Received May 28, 1934.)

234. Professor Morgan Ward: *Note on the iteration of functions of one variable.*

Let $E(x)$ be a real continuous single valued function of the real variable x in the range $a\leqq x<\infty$ such that $E(x)>x$, for all $x\geqq a$, and $E(x')>E(x)$, if $x'>x\geqq a$. An explicit formula is given for the continuous iteration of $E(x)$. (Received May 28, 1934.)

235. Professor Morgan Ward: *Note on an arithmetical property of recurring series.*

In this paper certain results by Siegel (Tôhoku Journal, 1921, pp. 26–31) oh the number of zeros appearing in a sequence of integers satisfying a recursion relation of order three are extended. It is shown that if the associated cubic equation has two complex roots, in general three such zeros can appear. A weaker result for sequences satisfying a recursion relation of order four is also established. (Received May 28, 1934.)

236. Professor D. W. Woodard: *Uniform set-theoretic characterizations for closed n-cells.*

A characterization of the closed n-cell has been given by Alexandroff in Mathematische Annalen (vol. 94, p. 296). In recent years several characterizations of the 2-cell have appeared. The word *uniform* as used in the title is justified in the sense that the main theorem as given may be regarded as a set of words and symbols defining a function, $F(n)$, such that $F(1)$ is a closed 1-cell, $F(2)$ is a closed 2-cell, and so on. The work is characterized by the use of the concept of *strong homeomorphism* due to Kline. A set M_1 is said to be strongly homeomorphic to a set M_2 provided there exists a topological transformation of $\overline{M}_1$ into $\overline{M}_2: H(\overline{M}_1)=\overline{M}_2$ such that $H(M_1)=M_2$. There is reason to believe that the concepts and methods used in the paper can be made to yield set-theoretic characterizations of at least certain classes of n-dimensional manifolds. In fact, they were selected with this result in view. (Received May 28, 1934.)

237. Professor Raymond Garver: *The effect of dumping on monopoly price.*

The subject of this paper was suggested by a similar paper by Professor Yntema, Journal of Political Economy (vol. 36 (1928), pp. 686–698). Its aim is to present a complete mathematical treatment of the problem. (Received May 29, 1934.)

238. Professor B. C. Wong: *Loci of m-spaces joining corresponding points of $m+1$ projectively related n-spaces in r-space.*

In this paper we generalize the theorem that the locus of the ∞^1 lines joining corresponding points of two projective point-rows in 3-space is a quadric surface and prove that the locus of the ∞^n m-spaces joining corresponding points of $m+1$ given projectively related n-spaces in an r-space where $r=mn+m+n$ is an $(m+n)$-dimensional variety of order $\binom{m+n}{n}$. The equations of this variety are derived and some of its properties are stated. The surfaces in which it is intersected by $n(m+1)$-spaces, in particular those for which $n=2$, are described. (Received May 29, 1934.)

239. Professor B. C. Wong: *Linear systems of hypersurfaces in space of r dimensions.*

In the equation $\sum_{i=0}^{\rho}\zeta_i f^{(i)}=0$ of a linear ∞^ρ-system $|V|$ of n-ic hypersurfaces in an r-space S_r where the ζ's are homogeneous parameters and the f's are homogeneous functions of degree n in $r+1$ variables, we let the ζ's be the coordinates of a point in a ρ-space S_ρ, and thus set up a one-to-one correspondence between the hypersurface of $|V|$ and the points of S_ρ. In this paper we are concerned with the varieties of various dimensions of S_ρ whose points correspond to those hypersurfaces of $|V|$ which are tangent to given varieties, in particular linear subspaces, of S_r, and describe some of their properties and their relations to one another and to the manifold whose points correspond to the Jacobian variety of $|V|$. Some particular cases, that is, cases $r=\rho=2$, n general; $n=2$, $r=\rho$ general; and $n=2$, $r=\rho=3$ are more closely studied. (Received May 29, 1934.)

240. Mr. A. E. Taylor: *On integral invariants of non-holonomic dynamical systems.*

It is well known that there are certain invariants associated with holonomic dynamical systems. Cartan has shown that a Hamiltonian system may be characterized by means of the relative integral invariant $\int_C \Sigma_i p_i dq_i - Hdt$. This paper extends the theory to non-holonomic systems. If the system has n degrees of freedom and $n+k$ natural coordinates, its equations of motion may be written in pseudo-Hamiltonian form, using a set of *Langrangian* multipliers. These equations are completely characterized by the relative integral invariant

$$\int_C \int_0^t \sum_{\nu=1}^{k} \lambda_\nu \left\{ \sum_{i=1}^{n} a_{\nu i} dq_i + a_\nu dt - dq_{n+\nu} \right\} dt + \int_C \sum_{s=1}^{n+k} p_s dq_s - Hdt,$$

where the a's are functions which enter as a result of the non-holonomic character of the system. (Received May 31, 1934.)

241. Professor E. V. Huntington: *The relation between Lewis's strict implication and Boolean algebra.*

This paper presents a proof of two new theorems in C. I. Lewis's system of strict implication, namely,

(1) $$(p \prec q) \prec [(p \prec pq)(pq \prec p)]$$

and

(2) $$[(p \prec pq)(pq \prec p)] \prec (p \prec q).$$

On the basis of these theorems it is shown that the relation of strict implication in Lewis's system is substantially equivalent to the relation called subsumption in ordinary Boolean algebra; that is, "p implies q" and "p within q" are interdeducible. (Received June 11, 1934.)

242. Professor M. H. Ingraham: *The non-singular case of the equivalence of pairs of Hermitian matrices.*

Consider two pairs of $n \times n$ Hermitian matrices (A, B) and (C, D). One pair is said to be equivalent to the other if there exists a non-singular matrix T such that $\overline{T}'AT = C$ and $T'BT = D$. There is no loss of generality if it is assumed that the rank of $\rho A + \sigma B$ never exceeds the rank of B. For the case where B is non-singular Mr. K. W. Wegner (see abstract No. 40-1-103) found necessary and sufficient conditions for the equivalence of the two pairs. This paper solves the general problem of equivalence by reducing the pair (A, B) to a canonical pair (A_1, B_1) in which B_1 is a diagonal matrix with diagonal terms limited to $1, -1$, or 0; and A_1 is a matrix of sub-matrices which is completely determined by the rank of all but one of these sub-matrices and the canonical form of this one for the non-singular case. (Received May 19, 1934.)

243. Dr. N. E. Rutt: *Prime ends and indecomposability.*

In this paper the following theorem is proved. If the boundary Γ of the plane bounded simply connected domain γ contains an indecomposable continuum D, there is a prime end of γ which contains D. (Received June 6, 1934.)

244. Professor E. T. Bell: *On the power series for elliptic function.*

In this Bulletin, vol. 26 (1919), pp. 19–25, the author gave incidentally certain expansions of elliptic theta quotients, in arithmetized form, derived from a comparision of the Fourier and MacLaurin expansions of the quotients. In the present paper (to be published in the Transactions of this Society) these expansions are applied to obtain explicit formulas for the numerical coefficients in the power series for elliptic functions. The formulas published without proof or indication of method by Hermite are derived very simply. (Received June 30, 1934.)

245. Dr. H. L. Garabedian: *An inclusion theorem in the theory of summable series.*

Sufficient conditions are obtained to insure that any definition of summability with infinite matrix of reference be more effective than or include the definition of de la Vallée-Poussin. For the sake of convenience we define a series $\sum_{n=0}^{\infty} u_n$ to be ϕ-summable to the sum 1 provided that the series $\sum_{n=0}^{\infty} \phi_n(s) u_n$ converges for $s>0$ and $\lim_{s\to 0} \sum_{n=0}^{\infty} \phi_n(s) u_n = 1$. The main theorem is stated as follows. *Suppose* (i) *that the series* $\sum_{n=0}^{\infty} u_n$ *is summable by the method of de la Vallée-Poussin to the sum* 1, (ii) *that* ϕ_n *is a function of* s *with the properties* (α_1) $\lim_{s\to 0} \phi_n(s) = 1$, (α_2) $\lim_{s\to 0} \nu^n \phi_n(s) = 0$, (α_3) $\sum_{n=0}^{\infty} \binom{2n}{n} \left| \sum_{\lambda=0}^{n} \binom{\lambda+2n}{} \Delta^{2n+2} \phi_{2\lambda+n}(s) \right| < K$, *if* $s>0$, *where* ν *and* K *are independent of* s *and* n, (iii) *that* ϕ*-summability includes Cesàro summability of arbitrary integral order. Then the series* $\sum_{n=0}^{\infty} \phi_n(s) u_n$ *converges if* s *is positive and* $\lim_{s\to 0} \sum_{n=0}^{\infty} \phi_n(s) u_n = 1$. (Received June 19, 1934.)

246. Professor E. V. Huntington: *The inter-deducibility of the new Hilbert-Bernays theory and Principia Mathematica.*

On account of the differences in notation (and in the choice of primitive ideas) between the calculus of propositions as developed by Whitehead and Russel in *Principia Mathematica* (1910, 1925) and the same calculus as developed by Hilbert and Bernays in their new *Grundlagen der Mathematik* (May, 1934), it may be a convenience to students of these two works to have at hand an explicit proof of the fact that each of these theories is deducible from the other. Parts I and II of the present paper supply the details of such a proof (not elsewhere available), thus showing explicitly that "implication" in the Hilbert-Bernays book is the same thing as "material implication" in the *Principia*. Part III shows that in the Hilbert-Bernays list of postulates the informal "rule of inference" can be proved to be formally "independent" by the same method that has hitherto been applied only to the formal postulates of the system. (Received June 29, 1934.)

247. Mr. W. H. Ingram: *Dynamics of multi-phase commutator motors.* Preliminary report.

In the case of the 3-phase salient-pole commutator, for example, the physical coordinates u_1, u_2, u_3 are transformed into true coordinates q_i by the equation $dq_i/dt = \sum (1/3 + (2/3) \cos (\theta - \beta - (i-j)2\pi/3))du_j/dt$, where $d\theta/dt$ is the angular velocity of the commutator and $d\beta/dt$ the angular velocity of the brushes. The equations of motion in the true coordinates are the same as for an orthocyclic slip-ring machine except that the external forces, given by the activity function expressed in the true variables, are non-palpable. Intermediate transformations to Park-Blondel coordinates permit of exact tests of stability but are otherwise not necessary. (Received June 27, 1934.)

248. Dr. E. R. Lorch (National Research Fellow): *A note on permutable self-adjoint transformations in Hilbert space.*

Let H_1, H_2 be two self-adjoint transformations in Hilbert space and let $E_1(\lambda)$, $E_2(\mu)$ be their respective resolutions of the identity. It is known that if H_1 is bounded, in order that the two transformations be permutable, i.e., (1) $H_1H_2 \subseteq H_2H_1$, it is necessary and sufficient that (2) $E_1(\lambda)E_2(\mu) \equiv E_2(\mu)E_1(\lambda)$. If H_1 and H_2 are both unbounded the definition of their permutability as given by Stone and von Neumann is contained in equation (2). In this note we in part justify this definition by proving that if (2) is satisfied there exists a self-adjoint transformation H_3 such that $H_3 \supseteq H_1H_2$, $H_3 \supseteq H_2H_1$. Furthermore the operators H_1H_2, H_2H_1, $H_1H_2 - H_2H_1$ are essentially self-adjoint (their closed linear extensions are self-adjoint). (Received July 6, 1934.)

249. Mr. Haim Reingold and Professor I. A. Barnett: *Seminvariants of a system of linear homogeneous differential equations of the second order.*

The system of equations in question is as follows: $d^2y_i(x)/dx^2 + \sum_{j=1}^n L_{ij}(x)dy_i(x)/dx + \sum_{j=1}^n M_{ij}(x)y_j(x) = 0$, $(i = 1, 2, \cdots, n)$. It is proposed to find the expressions involving L_{ij}, M_{ij}, $L'_{ij} \equiv dL_{ij}/dx$, which remain unchanged by the transformations $y_i(x) = \eta_i(x) + \sum_{j=1}^n K_{ij}(x)\eta_j(x)$, $(i = 1, 2, \cdots, n)$; where the $K_{ij}(x)$ are arbitrary. By the introduction of a certain matrix G_{ij} depending upon the matrices L_{ij}, M_{ij}, L'_{ij}, it is found possible to exhibit a complete set of seminvariants which turn out to be the n successive traces of the powers of G_{ij}. Two incidental results are worth noting. One, a lemma used in the proof of the main theorem of this paper, states that the partial derivative of the trace of the lth power of a matrix with respect to any element is equal to l times the corresponding element of the $(l-1)$th power of the transposed matrix. The second result, which is a corollary of the main theorem, is that a necessary and sufficient condition that the matrix obtained by differentiating any function with respect to a_{ij} be commutative with the transposed matrix a_{ji}, is that the function be of the form $f(a_{ii}, a_{ii}^{(2)}, \cdots, a_{ii}^{n})$, where a_{ii}^k is the trace of the kth power of a_{ij}. (Received June 21, 1934.)

250. Dr. G. E. Schweigert: *The analysis of certain curves by means of derived local separating points.*

For any connected subset M of a compact, metric, hereditarily locally connected continuum H, let $L_0(M)$ denote the set of all local separating points of M. The set $H - L_0(H)$ consists of the derived components $[C_i^1]$ of index 1 and a zero-dimensional set P_1. The derived local separating points of index 1 are defined to be the points in $L_1(H)=\sum L_0(C_i^1)$. In general $L_\alpha(H)=\sum L_0(C_i^\alpha)$ are the derived local separating points of index α, where the C_i^α are the derived components of index α in the set $H-\sum L_\beta(H)$, for $\beta<\alpha$. The index of the curve H is the least transfinite ordinal $\alpha(H)$ such that $L_{\alpha(H)}(H)=0$. In this paper it is shown that if p is a point of C_i, then $g(p)\geqq\alpha$ where $g(p)$ denotes the genus of p in the sense of Menger. It follows that the genus $g(H)\geqq\alpha(H)$ and hence that $\alpha(H)$ is of the first or second number class. The set $H-\sum L_\beta(H)$, for $\beta<\alpha(H)$, is of dimension zero. It is shown that H may be dispersed equally well by a countable number of the derived local separating points. (Received June 12, 1934.)

251. Mr. P. L. Trump: *On a reduction of matrices by the group of matrices commutative with a given matrix.*

Given an $n\times n$ matrix A, we seek to determine conditions under which two $n\times n$ matrices B and C, commutative with A, are in the relation $B=S^{-1}CS^{(1)}$ where S is an element of the group $[S]$ of non-singular $n\times n$ matrices commutative with A. The problem easily reduces to a consideration of the case in which A is assumed to be in classical canonical form with characteristic roots all zero. The work is simplified by establishing an isomorphism which leads to the consideration of matrices P, of reduced dimension, whose elements $P_{ij}(\gamma)$ are polynomials in γ of a restricted type determined by A. A reduction to possible canonical forms is attempted and is successful to the extent of reducing to a consideration of matrices P in which the elements along the main diagonal are congruent to each other modulo γ. This case is now under consideration. Procedures are outlined which, in any case, establish the possibility of determining whether or not two particular matrices of type commutative with A are in relation (1). (Received July 6, 1934.)

252. Professor W. A. Wilson: *On certain types of continuous transformations of metric spaces.*

Let $y=f(x)$ be a continuous transformation of one metric space X into another Y, let a be a limiting point of X, let x and x' be distinct points of X, and let y and y' be the corresponding points of Y. If $\lim\ (yy'/(xx'))$ exists, finite or infinite, as x and x' approach a, this limit is called the spread of $f(x)$ at the point a. Various properties of the spread of a continuous transformation are found and applications to the rectifiability of arcs and allied topics are given. If the transformation is such that to each distance u in X there corresponds the distance v in Y and the correspondence $v=\phi(u)$ is a continuous function defined for $u\geqq 0$ and satisfying the following conditions: $\phi(0)=0$; $\phi(u)>0$ if $u>0$; and $\phi(u_1)+\phi(u_2)\geqq\phi(u_3)$ if $u_1+u_2\geqq u_3$, the transformation is called regular. It is shown that regular transformations are homeomorphisms, that $\phi'(0)$ exists and is greater than zero, and that $\phi'(0)$ is the spread of $y=f(x)$ at every limiting point of X. With a proper definition of angles, it is found that, if $\phi'(u)$ exists and is continuous at $u=0$, various types of smoothness of simple arcs are preserved, and the transformation is conformal. In the special case that the inverse of a regular transformation is also regular, the transformation is one of similitude. (Received July 2, 1934.)

253. Professor W. A. Wilson: *On the imbedding of metric sets in euclidean space.*

Two results are obtained in this note. The first is that a convex complete metric space which has the four-point property also has the n-point property for every integer n. This removes the condition of external convexity imposed in a former proof by the writer. The second is that Menger's conditions for the imbedding of n points of a metric space in euclidean space are equivalent to the requirement that for any $k+3$ of these points, where $1 \leqq k \leqq n-3$, determining three k-dimensional half-spaces having in common the $(k-1)$-dimensional space determined by some sub-set of k of these points, the sum of the three space angles having these half-spaces as "sides" and the common space as a "vertex" is not greater than 2π and their values satisfy the metric triangle inequality. These angles are readily calculated by the plane and spherical cosine laws. (Received June 14, 1934.)