SHEWHART ON ECONOMIC CONTROL

Economic Control of Quality of Manufactured Product. By Walter A. Shewhart. New York, Van Nostrand, 1931. xiv+501 pp.

This important book, written by a recognized authority in the fast developing field of mathematical statistics, comes as a welcome addition to the all too sparse collection of expository books. Those who have wished in their statistical courses for practical problems to illustrate sampling theory, the correlation surface, frequency distribution of parameters such as the mean and standard deviation, and the use of Chi Square, will be enthusiastic about the splendid collection of practical problems most of which have their origin in the telephone business.

It has been known for some time (at least since the time of Gauss) that the distribution of the average $\overline{X}$ of a sample taken from a normal universe is normal with a standard deviation $\sigma/\sqrt{n}$. Student, Karl Pearson, and R. A. Fisher have studied the distribution of the standard deviation, and Fisher has given the theoretical formula. For non-normal universes we do not know the distribution of so simple a statistic as the arithmetic mean. Therefore, for non-normal universes appeal must, at present, be made to experimental data. Dr. Shewhart presents observed distributions of average of 1,000 samples of four taken from (1) a rectangular universe and (2) a triangular universe. These distributions appear to approach normality as the number of samples increases and as a result it seems safe to say that in almost all cases in practice we may establish sampling limits for averages of samples of four or more upon the basis of normal law theory. Shewhart gives also the results of experiments to obtain the distribution of the standard deviation when the universe is not normal. The summary of available information in respect to some of the more important statistics, which is given on page 212, should be of challenging interest to mathematicians.

After briefly indicating in a general way in Chapters 1, 2, 3, and 4 how it is possible to use modern statistical theory to control quality of the manufactured product, Dr. Shewhart digresses to give a practical presentation of statistical theory, especially the modern theory of sampling. Some elementary but important problems of presenting data by tables and graphs are considered in Chapters 5 and 6. Such statistical concepts as arithmetic mean, median, mode, standard deviation, skewness, kurtosis, correlation coefficient, and correlation ratio are defined and their calculation is illustrated in Chapter 7. A discussion of the relative usefulness of these statistics and a development of Tchebycheff's theorem are given in Chapter 8. A study of correlation and relationship is presented in Chapter 9. Laws basic to the control of quality, that is, the law of large numbers, the point binomial, and the meaning of statistical laws are described in Chapter 10. The development of statistical concepts will be found to be sufficiently complete for those who do not require the details of difficult proofs. Those who wish to investigate proofs will find ample references to periodical literature. An excellent bibliography is given as Appendix III.

Chapters 11 to 21 can best be described by presenting outlines of some of the problems of control.

At present we are coming more and more to realize that we cannot do things just as we would like to do them. We can, however, approach a state of perfection by eliminating known causes of variation from standard. Thus, we may have control within limits. In Dr. Shewhart's words: "since we are thus willing to accept as axiomatic that we cannot do what we want to do and cannot hope to understand why we cannot, . . . we must also accept as axiomatic that a controlled quality will not be a constant quality." "A phenomenon will be said to be controlled when through the use of past experience . . . we can state, at least approximately, the probability that the observed phenomenon will fall within the given limits."

If only chance causes affect the quality of a product, then the probability dy of the magnitude X of a characteristic of the quality falling within the interval $X+dX$ is expressible as a function f of the quantity X and certain parameters represented symbolically in the equation $dy=f(X, \lambda_1, \lambda_2, \cdots, \lambda_m)dX$. Dr. Shewhart calls such a system of causes *constant*, because the probability dy is independent of time. Thus, if a cause system is not constant, there is an *assignable cause* present.

"Stated in terms of effects of a cause system, it is necessary that differences in the qualities of a number of pieces of a product *appear* to be consistent with the assumption that they arose from a constant system of chance causes." If a product is statistically controlled, that is, if the system of chance causes appears to be constant, "there is an objective probability p that a piece of this product will be defective." According to the law of large numbers "the observed fraction defective in successive samples of size n should be clustered or distributed about the value $p=P$ in accord with the terms of the point binomial $(P+q)^n$."

In practice there will be a sequence of observed values of the fraction defective p. The method of attack for determining whether or not the quality as measured by fraction defective is statistically controlled is to establish limits of variability of p such that when a fraction defective is found outside these limits, looking for an assignable cause is worthwhile. Of course, no matter how these limits are set so long as they are not outside the limits of the frequency distribution, some of the observed fraction will fall outside the limits. Hence if trouble is looked for each time p falls outside the limits, there will be times when no trouble can be found. The practical problem, therefore, requires that the limits be *economic*, that is, a balance must be struck between the advantages to be gained by reduction in the cost of looking for trouble when it does not exist and the disadvantages occasioned by overlooking troubles that do exist.

Tchebycheff's theorem provides a theoretical way for setting limits of quality. This theorem states that the probability, P, that a statistic θ will lie within limits $\bar{\theta} \pm t\sigma_\theta$, where $\bar{\theta}$ is the expected value of θ and σ_θ is the standard deviation of θ, satisfies the inequality $P>1-1/t^2$. According to Dr. Shewhart, experience indicates that $t=3$ is an acceptable economic value. Dr. Shewhart's method for establishing allowable limits of variation in a statistic θ therefore depends upon theory to furnish the expected value $\bar{\theta}$ and the standard deviation σ_θ of the statistic, and upon empirical evidence to justify the choice of limits $\bar{\theta} \pm t\sigma_\theta$.

Statistics which are most useful in control of quality are the mean and standard deviation. Skewness and kurtosis are of little if any value for this problem. Hence from a design viewpoint the specifications of control should include the specifications of expected value X_i and standard deviation σ_i of any quality characteristic X_i. For such specifications it is possible to test for control by constructing charts for the mean $\overline{X}$ and standard deviation σ using as boundaries $\overline{X} \pm 3\sigma_x$ and $\bar{\sigma} \pm 3\sigma_\sigma$. If a value of either X or σ for a sample falls outside these limits, this may be taken as an indication of lack of control.

Another important problem is that of determining if it is probable that samples came from the same system of chance causes. For this problem neither the mean $\overline{X}$ nor the standard deviation σ are known and it becomes necessary to estimate them from the samples themselves. Dr. Shewhart points out that it is desirable to choose estimates that will be most likely to indicate lack of constancy in the cause system, and proposes that σ be chosen by means of a theoretical formula $\bar{\sigma} = c_2\sigma$, where $\bar{\sigma}$ is the expected value of σ and c_2 is a constant for a given sample size. He gives tables of this constant for values of sample size from 3 to 100. Once estimates of $\overline{X}$ and σ are available, the problem is essentially that already described.

Chapter 21 is concerned with the problem of detecting the presence of a predominating cause or group of causes forming a part of a constant system. Three methods of detection are given. The first involves the use of the correlation coefficient, the second of the Tchebycheff inequality and theoretical knowledge of the distribution of means and standard deviations, and the third of Chi Square.

Chapter 22 is devoted to a general discussion of modern industrial research, physical properties of matter, and various means of estimating average and standard deviation. Here a short discussion of the method of maximum likelihood is given. Chapter 23 considers the relationship of measurement to sampling. Chapter 24 treats methods of sampling and size of sample. Chapter 25 contains a general résumé of control and reasons for control.

This book is on the whole very well written and contains very few errors. The style is interesting for those who like variety of illustrations and consequent repetition. The statistical reader will do well to read the introductory chapter and then turn to Chapter 11.

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