Elementary Mathematics from an Advanced Standpoint. Arithmetic, Algebra, Analysis. By Felix Klein. Translated from the third German edition by E. R. Hedrick and C. A. Noble. New York, Macmillan, 1932. 10+274 pp.

Klein possessed in an unusual degree the abilities of a great mathematician and the gifts of an inspiring teacher and lecturer. He had a broad knowledge of mathematics and a correspondingly deep insight into the foundations and interrelations of its various branches. Both Klein's qualifications for writing a book of this nature and the scarcity of such books combine in directing attention to the present volume.

This book, a translation of the first of Klein's three volumes entitled *Elementarmathematik vom höheren Standpunkte aus*, is a series of lectures that Klein gave for teachers of mathematics in secondary schools. The material is presented under the headings of arithmetic, algebra, analysis, and a supplement. The section on arithmetic treats the extensions of the number system and the laws of operation, beginning with integers and ending with complex numbers and quaternions. The treatment seeks to explain the *how* and *why* of the subject. As an example, we note the discussion of the little understood rule of signs: "minus times minus gives plus." The section on algebra is devoted to the solution of equations. First, some geometric methods are explained for investigating the real roots of rational integral equations containing parameters. Then complex roots are considered, especially of those equations whose solutions lead to a consideration of the groups of motions connected with the regular bodies. Free use is made of Riemann surfaces and other parts of the theory of functions of a complex variable. The section on analysis is devoted to the logarithmic, exponential, and trigonometric functions, and a discussion of the infinitesimal calculus proper. A wide variety of subjects is treated, however, in connection with these general topics: the construction of the early logarithmic and trigonometric tables, expansions in Fourier series, Taylor's Theorem, and Newton's and Lagrange's interpolation formulas will serve as samples. Finally, the supplement contains proofs of the transcendence of e and π, and a discussion of assemblages.

The real excellence of the book, however, is due to certain clearly defined characteristics of the presentation. In the first place, the historical development of the theory is traced. This is not history for history's sake alone, but history as an aid to gaining a deeper insight into the present state of the theory. In this connection it should be stated that the inductive method of presentation is used exclusively.

Secondly, the geometric aspects of the subjects treated are emphasized. It is significant that the book contains 125 figures. The geometric meaning of Fermat's Theorem is explained; the Pythagorean number triples are obtained by a geometric method. The graphs of the approximating polynomials of Taylor's series expansions are drawn in order to show the nature of the convergence and divergence; similarly for Fourier series. Klein would develop geometric intuition and sense perception as an aid to mathematical investigation.

Again, Klein shows the mutual relations between problems in different fields. His ability to discover such relations is well known, and many examples are to be found in this volume. We may mention the parallel treatment of the logarithmic, trigonometric, and hyperbolic functions, one which incidentally robs Euler's formula, $e^{i\theta} = \cos\theta + i\sin\theta$, of its mystery.

Finally, the book is in the form of lectures. Somewhat informal, it is all the more vigorous and interesting. We hear Klein as he addresses his class, explains a drawing or a model, or hands them an early book for examination; we feel the inspiration that comes from listening to a great teacher.

The paper, printing, and binding are of high quality. The reader will notice a number of minor typographical errors, but they are largely misspellings of words that do not cause any inconvenience.

This book is familiar to many already in the original, and this review is written in the hope of introducing it to a still wider circle of readers among our students and teachers of high school and college mathematics. Our thanks are due to Professors Hedrick and Noble, who have rendered a service to mathematics by making this valuable book accessible to all who speak English.

G. B. PRICE

L'Hydrodynamique et la Théorie Cinétique des Gaz. By Y. Rocard. Paris, Gauthier-Villars, 1932. 160 pp.

The fluid of hydrodynamics is a continuum, while the kinetic theory views a fluid as formed of discrete molecules. Maxwell, Boltzmann, and Lorentz laid the foundation for bringing these two different methods of approach into harmony, and, more recently, their work has been carried on by Brillouin, Chapman, Enskog, and others. The two principal problems are to obtain the equations of hydrodynamics for real fluids from the kinetic theory, and to calculate the coefficient of viscosity by considering the molecular encounters.

The author has made important contributions to this subject himself, and has performed a very useful service in collecting into a single volume the results of all these investigations. This book is based upon lectures by the author in 1929 and upon a memoir published in the Annales de Physique in 1927.

E. P. ADAMS

Caractéristiques des Systèmes Différentiels et Propagation des Ondes. By Tullio Levi-Civita. Leçons rédigées par G. Lampariello. Traduction de l'italien par M. Brelot. Paris, Librairie Félix Alcan, 1932. x+114 pp.

In 1930 Professor Levi-Civita published two papers on the characteristics and bicharacteristics of Einstein's gravitational equations, and it was probably during the preparation of these papers that he realized the need of a clear and concise account of the theory of characteristics such as is given in his little book in the Italian language which was reviewed in this Bulletin of May, 1932.

Judging from the papers of Racah and Lampariello, Levi-Civita's work has been very stimulating to Italian mathematicians and so it is good news to find that his lucid and timely exposition has been made available to a greater number of readers by M. Brelot's excellent translation.

The use of large print for names of authors, heavy type for vectors, and a familar notation, all make the book very readable.

H. BATEMAN