SHORTER NOTICES

Die mathematische Denkweise. By Andreas Speiser. Zurich, Rascher, 1932. 137 pp.+9 pp. music.

This book is not a treatise on how mathematicians think. It is a collection of essays on mathematical thought as it is revealed in art and music, in philosophy and astrology. It is the work of a man of broad culture—one whose contributions to group theory are well enough known, but who is also at home in yet more esthetic realms and is conversant with the history of serious human thought.

The first chapter, *Delimitations*, while not attempting to define mathematics, sketches the history of its relations (in the few centuries preceding 1900, relations of decreasing intimacy) with philosophy and art, as well as with language, physics, and religion. There is good history, sound wisdom—a foretaste of a rewarding book. It is the symmetry and rhythm in art which is most closely akin to mathematics and especially to the mathematics of groups. (One might, of course, prefer to say that mathematics is artistic.) The love for such symmetry, for a neat formulation, sometimes leads Speiser to statements which are more suggestive than convincing. Thus, the triadic form of Proclus and Hegel is used to characterize center, radius, and circumference of a circle, and then: "Mathematics forms a triad with philosophy and art, wherein philosophy corresponds to the inert center, mathematics, as progressive research, to the *πρόοδος*, art as limiting and formative activity, which attains beauty by being driven back upon the center, to the *ἐπιστροφή*."

Two chapters are next devoted to *Symmetries in ornamental art* and *Questions of form in music*. In the former there are examples from many stages of history (unfortunately it was financially impossible to give the illustrations which would have facilitated the argument). In the latter the symmetries in simpler musical forms are discussed, and here we do find a number of themes reproduced, and a considerable portion of a Beethoven sonata analyzed. That symmetry does appear here and contributes greatly to the beauty of the work is undeniable. Yet, were that quality sufficient, we could look forward to a machine for the production of masterpieces. In fact, symmetry, in the strict mathematical sense, is intolerable. Speiser recognizes this in describing musical performance. "The nuances are by no means gratuities of sensitive souls, but the main pillars of the reproduction of a piece, since they alone reveal its symmetric content. The extraordinary accuracy which artist and listener bring into play is one of the clearest proofs of the mathematical structure of the human spirit"—but is not the need for these very nuances also a proof that mathematics is not enough? An amusing hypothesis is this—"perhaps the good work of art is characterized by a minimal property; it is the simplest piece consistent with the symmetry complex which it contains." There is another aspect of the affinity of mathematics with music, which Speiser might have mentioned. The satisfaction coming from these two arts is, in part, due to that temporal element which is lacking in visual art. A musical composition, a mathematical proof, needs for its greatest esthetic effect a sense of suspense during its development, with the full resolution deferred until the end. If all good things come by threes, we might add the detective story as the third example of this type.

The later chapters deal with philosophy: Dante's philosophy of nature, Proclus Diadochus (the most admired of the ancient mathematicians) on mathematics, number and space with the Neo-Platonists, Goethe's color doctrine, astrology. In all these essays we have an author who has the advantage (the word of course reveals prejudice) of the newer empirical point of view, and yet gives most sympathetic accounts of those idealists who believed that cogitation would disclose the truth about reality. We have, for example, Dante's explanation of why all inhabited land should be in the form of a lunette—with its center at Jerusalem—exactly opposite the divine point of heaven. Originally the Godhead attracted the bulk of solid matter to the opposite side of the sphere. But "Lucifer plunged headlong down from this divine point. When the earth saw him coming, it shrank back aghast. Lucifer was held fast in the center, and above his three heads there appeared a cave, Hell; then earth closed behind him again, and thus there arose the Mountain of Purgatory near the Antipodes." If it is a little less than fair to cite only this example of the working of reason uncontrolled by experiment, we would atone for it by urging that the book itself be read. It is not a necessity for one's library; it is a delight.

E. S. Allen

Lezioni sulla Teoria delle Superficie Algebriche. By Federigo Enriques. Raccolte di Luigi Campedelli. Parte 1a. Padova, Antonio Milani, 1932. 4+481 pp.

The book contains 481 pages and comprises the first part of a volume assembled by Dr. Luigi Campedelli in collaboration with the author. It is not made clear, however, whether the work is presented as a preliminary edition of a text or as a set of organized lecture notes.

A general introduction of twenty pages embraces birational transformations, examples of singularities, geometry upon surfaces, and exceptional curves. Among this material occurs the fundamental theorem to the effect that a surface F, having any singularities whatever, may always be transformed, by means of a birational transformation, into another surface having no singularities.

Without listing the actual contents it may be said that there are five chapters embracing sixty-six paragraphs. The chapter topics are: systems of linear curves, systems of covariants and invariants, adjoint surfaces, the genus number and the theorem of Riemann-Roch, classification of surfaces, in particular, regular surfaces.

Chapter 1 contains illustrative examples of the topics under discussion; the same technique is observed in Chapter 3; Chapters 2, 4, 5 have appended bibliographies. Chapter 5 embraces more than one third of the book and includes an extended discussion of canonical surfaces and curves, Cremona transformations, rationality of surfaces and planes in connection with the theorems of Noether and Castelnuovo.

The book has apparently been photostated; the title, chapter headings, etc., are hand printed while the body of the text is in excellently executed script.

H. R. Phalen