

Where Is Science Going? By Max Planck. Prologue by Albert Einstein. Translation and Biographical Note by James Murphy. New York, W. W. Norton and Co., 1932. 221 pp. Price, \$2.75.

A great scientist and great man here analyzes the present state of physics and a half century of its development with reference to its implications for the further progress of science and for certain major interests of human welfare in general. The prologue by Einstein, discussing the motives which impel some of our choicer spirits to scientific research, furnishes a fitting prelude to the general theme of the book. The introduction by Murphy will stimulate the reader's interest in Planck both as scientist and as man. Chapter I, *Fifty years of science*, (pp. 41-63), gives a brief non-technical but illuminating presentation of the background of the present state of physics. The second chapter argues for the reality of the external world and sharply opposes a bare positivistic philosophy of science. This reality can never be fully attained; but "it is not the possession of truth, but the success which attends the seeking after it, that enriches the seeker and brings happiness to him." The third chapter is devoted to the scientist's picture of the physical universe. The next two chapters treat of causation and the freedom of the will. It is too much to expect any one to deal satisfactorily with the latter problem. With respect to the former the considered judgment of Planck is that the principle of causality should be given such a modified formulation as to restore it to strict validity. The final chapter deals with the passage from the relative to the absolute. The book closes with an epilogue in the form of a Socratic dialogue by Planck, Einstein and Murphy. Throughout this stimulating volume the treatment is as free of technicalities as the nature of the subject admits.

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Kurventheorie. By Karl Menger. Herausgegeben unter Mitarbeit von Georg Nöbeling. Berlin and Leipzig, B. G. Teubner, 1932. vi+370 pp.

In the last chapter of Pierpont's *Theory of Functions of Real Variables*, vol. II (1912), there is a discussion of several geometric concepts, including that of a plane curve. Various definitions which might appear reasonable at first sight are examined and shown to be inadequate, with the net result that the question, "What is a curve," is not answered. After the lapse of twenty years, we have a book devoted to this question, which gives the answer and discusses extensively the properties of curves, in a way little dreamed of in 1912.

The book under discussion is to be the second volume of a work entitled *Mengentheoretische Geometrie*. In addition to being the first treatise on the theory of curves and their topological properties, the book performs the further service of gathering together in an organic whole much of the research work on this subject done since the war—work which, from the nature of its piecemeal appearance, must have seemed to many to be of a pointless character. To Americans this book is of special interest, in view of the generous acknowledgment of their contributions to the subject in the publishers' announcement and in the bibliographical notes.

In the first chapter are given elementary notions from point-set theory for later use, and brief discussions of various types of point sets, such as irreducible

continua, locally connected continua, simple arcs, ordinary curves, and Cantorian curves—all of which are shown to be too narrow or too broad to serve as a definition of the term *curve*. Chapters II and III contain the definition of a curve essentially as a continuum one-dimensional at each point, and theorems on the structure of the sub-sets of points of various orders. In Chapters IV, V, and VI we have theorems on sums of curves, covering theorems, separation theorems, the n -Beinsatz, and the n -Bogensatz. The remaining six chapters deal with hereditary local connectivity, regular curves, rational curves, trees, cyclic continua and cyclic elements, and the universal curve. Finally we have a summary of the principal theorems, mention of topics not treated, and a brief discussion of certain unsolved problems.

Since the name of the author guarantees the excellence of this book, the reviewer feels that he can do little more than commend it to students of topology as indispensable and to prospective writers of mathematical treatises as a model of exposition, and notice certain special features. One of these is that, since the curve theory is in some aspects merely a discussion of 0-dimensional and 1-dimensional sets, the book will be of assistance to students reading the corresponding more general work in the *Dimensionstheorie* for the first time. The *Beispiele*, which include not only *examples* in the usual sense but also related bits of theory, and the references at the end of each section, are obviously of great value. Although, as the author points out, there is no essential loss of generality in supposing curves to be imbedded in euclidean space, the reviewer feels that in some cases proofs would appear less cumbersome if the common practice of regarding the curve itself as a compact metric space were followed. There are fewer instances where the continuity of thought is broken by the insertion of lemmas in the proofs of theorems than in the *Dimensionstheorie*. In one or two cases it appears that present or future workers ought to be able to find simpler demonstrations. (See, for example, L. Zippin, *Annals of Mathematics*, vol. 34, p. 95.) The choice of material in such a book as this is largely governed by physical limitations and the author's own work; it may, however, be worth while to note the small space allotted to linear graphs, external properties of curves, and the structure and decomposition of irregular curves. No systematic search for slips in reasoning and misprints has been made, but a certain amount of sampling indicates that these can be but few in number. Last, but not least, there is an excellent index and it is hoped that this will be a feature of any revision of the *Dimensionstheorie*.

In conclusion it may be remarked that, although it is often said that the appearance of a book on a subject indicates that its life as a field of research is nearly ended, there are reasons for thinking that this book may be a stimulus to further discoveries, both because of its rendering much of the work of the past accessible as a whole and because of the unexplored territory indicated by the author himself.

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