VOLUME I—ERGEBNISSE DER MATHEMATIK

Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 1, 1932.

Knotentheorie. By K. Reidemeister, 74 pp., 114 figures.

Graphische Kinematik und Kinetostatik. By K. Federhofer. 112 pp., 27 figures.

Lamésche-, Mathieusche- und verwandte Funktionen in Physik und Technik. By M. J. O. Strutt. 116 pp., 12 figures.

Die Methoden zur angenäherten Losung von Eigenwertproblemen in der Elastokinetik. By K. Hohenemser. 89 pp., 15 figures.

Fastperiodische Funktionen. By H. Bohr. 96 pp., 10 figures.

This series of reports forms a welcome addition to the work of abstracting published by the Zentralblatt für Mathematik and is being edited by the directors of this journal. The need of such reports is becoming more and more evident as scientific knowledge expands and it is particularly pleasing that this volume contains an astonishing amount of information in a compact form and presented in good style. The reports deal mainly with recent work and so supplement the articles in the Encyklopädie der Mathematischen Wissenschaften in which the history of each subject is given.

A reader who compares Reidemeister's modern account of the theory of knots with the early accounts given by Listing and Tait will realize that the subject has been greatly transformed by the use of groups, matrices, quadratic forms, and polynomials. A knot is regarded here primarily as a skew polygon. This plan simplifies the description of deformations and operations; it also facilitates the drawing of a regular projection of the knot, on a plane P. Such a projection divides P into regions which may be divided into black and white regions by a suitable method of coloring. Special names are used when there are only two or only three black regions.

The study of topological invariants, such as torsion numbers, forms an important part of the subject and it is interesting to note the substantial contributions which have been made by the American writers J. W. Alexander and G. B. Briggs. Reidemeister's report is well written and unlike many recent articles on topology does not make the subject look formidable.

Federhofer's report shows that pure geometry is still quite a live subject capable of interesting and useful applications in the study of driving mechanisms and linkages. The author has been particularly active in inventing new constructions for the elucidation of the motion of a plane system in a plane and the motion of a rigid system in space. He gives a fine account of the theory of kinematic chains and linkages, calling attention to the forgotten work of Grübler. The whole report is, indeed, furnished with very complete lists of references.

In kinetostatics the chief aim is to determine the guiding forces, linkage pressures and internal stresses, for arbitrary cross-sections, of the linked members of a chain of rigid bodies. The methods used are again geometrical but the masses of the members have to be taken into consideration. The analysis is due largely to Wittenbauer and to Federhofer himself. A second fundamental problem is to determine the force P_Q, acting at a definite place Q in a definite direction, which must be added to given impressed forces to produce a prescribed state of acceleration of a movable system of known mass distribution.

The name of Lamé has become associated with certain functions satisfying a linear differential equation of a special type in which the coefficient of the dependent variable is a doubly periodic function. These functions are related to the ellipsoidal harmonics and include many important functions as particular cases. When the ellipsoid is replaced by an elliptic cylinder, and Laplace's equation by the equation of wave-motion, the corresponding functions are those of Mathieu. The differential equation then has a simply periodic coefficient and is in fact identical with an equation studied by Laplace in 1777 when he explained his method of successive approximations. A report on these and allied functions is particularly timely now that the applications of such functions are so numerous and it is fortunate that the report has been written by a man like Strutt who is engaged in industrial research and is familiar with many of the important applications. The report is well written and contains much new matter.

Hohenemser's report is intensely practical and will be eagerly read by men who are not satisfied until they have obtained numerical results. The problem of elastic vibrations is at once formulated with the aid of integral equations and a good sketch is given of methods of iteration for the determination of the normal vibrations. As an alternative to a successive derivation of these, there is a method of van den Dungen, by which some of them may be obtained simultaneously. Problems of elastokinetics are next formulated as variation problems and, after examples have been indicated of the use of Rayleigh's principle, an account is given of its elaboration by Ritz, Galerkin, Kryloff, and others. In this work the approximations are made by functions which do not satisfy the differential equation (or equations). An alternative method, developed recently by Bergmann, Trefftz, and Friedrichs, employs functions which satisfy the differential equation but do not satisfy (all) the supplementary conditions. Some limitations of this method are pointed out.

The second part of the report describes methods depending on the use of linear differential equations such as those of Liouville and Sturm. The method of successive approximations is explained and an approximate method of calculating the frequencies of vibration of composite systems is indicated. Some remarks are made also on the solution of frequency equations expressed in determinant form. The report closes with a discussion of the numerical results obtained in special problems relating to the torsional and transverse vibrations of rods, the critical speeds of revolving shafts, and the vibration of plates. The report condenses the results obtained by wide reading, many of the journals being inaccessible to the general reader.

Harald Bohr expounds a branch of analysis which has been created almost entirely during the last few years. The preliminary chapter on purely periodic functions is characterized by a freshness of treatment in which emphasis is laid on the uniqueness theorem, the Parseval relation, and the theorem of Fejér. The extent to which one of these theorems is a consequence of another is indicated and much use is made of an extension of Parseval's theorem which involves the operation of folding and is important in the theory of almost periodic functions. The main theorem which is finally proved at the end of the lectures is that the class of almost periodic functions is identical with the class $H\{s(x)\}$ of functions which can be uniformly approximated by finite sums $s(x)$ composed of terms of type $a_n \exp(i\lambda_n x)$, where the coefficients a_n are complex quantities and the exponents λ_n real quantities all of which can be chosen freely. The analysis deals largely with mean value theorems, the multiplication theorem, the uniqueness theorem and its equivalence to the Parseval relation, limiting forms of integrals, and a convergence theorem for infinite series of type $s(x)$ when the Fourier exponents are linearly independent. The report closes with an account of some generalizations of the idea of an almost periodic function. The work is all of a high standard and will stand the scrutiny of mathematicians who insist on a statement of all the saving restrictions.

H. BATEMAN

KAMKE ON PROBABILITY

Einführung in die Wahrscheinlichkeitstheorie. By Erich Kamke. Leipzig, S. Hirzel, 1932. vii+182 pp.

This excellent *Theory of Probability* is an elaboration of the author's paper *Über neuere Begründungen der Wahrscheinlichkeitsrechnung.** In this paper Kamke sets forth a new foundation for probability and gives a critique of some closely related theories, those of R. von Mises, Dörge, and Tornier. Von Mises—whose work marks the beginning of a new era in probability theory—discards the conception of "equal likely events" as a proper basis for probability, in favor of a treatment based upon infinite sequences. If a die is thrown, and the results recorded,—for example, 2, 5, 3, 3, 6, 1, 4,—a sequence is obtained which may be thought of as continuing indefinitely. Now let m be the number of times that the five-spot appears in the first n throws. Some authors naively write: lim $m/n=1/6$. But, in general, no such limit exists. We know nothing about the future behavior of the die. A rigorous treatment of probability must divorce itself from all physical considerations. This the newer theories do. We may write: $a_1, a_2, \cdots, a_n, \cdots$, as symbols for a set of numbers, restricted to the values 1, 2, 3, 4, 5, and 6. And among all possible sequences of this description we may *choose to consider only* those for which lim $m/n=1/6$. And the properties of such sequences may be ascertained with mathematical rigor. Sometimes the sequence definition is called the "statistical" definition of probability. This is unfortunate, for the word "statistical" is likely to carry a reader's mind to the loose "limit" first mentioned. A sharp distinction between such a *pseudo-limit* and a real limit is fundamental for all clear thinking in probability. The theory of R. von Mises is grounded upon two axioms; that is, he *considers* only those sequences for which both axioms are valid. The first axiom relates to the existence of limits, such as has just been illustrated. The second axiom involves a selection—from the original sequence—of terms by some scheme which relates to their position or order—to the subscript r of a_r—

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