some properties of the elliptic modular functions, then yield the classical theorems of Picard, Landau, and Schottky.

In Chapter 5 the author introduces the notion of convexity of functions and derives the fundamental theorems of Hardy and Littlewood concerning the means of order p of analytic functions. In connection with Theorem IV of this chapter, a somewhat analogous theorem of F. Hausdorff may be mentioned. The book ends with a short account of F. Riesz's theory of subharmonic functions and its principal applications to the integral means of analytic functions.

Of the misprints in the book there is one of sufficient importance to warrant mention. In the statement of Lindelöf's fundamental theorem the hypothesis $z_0=f(x_0)$ (using the author's notation) is omitted.

The book serves the excellent purpose of unifying by means of geometric concepts various branches of the theory of functions which have hitherto been scattered in the literature. The presentation throughout is lucid, rigorous, and elegant.

W. SEIDEL

Richard Dedekind. Gesammelte mathematische Werke. Herausgegeben von Robert Fricke, Emmy Noether, Öystein Ore. Dritter Band. Braunschweig, Vieweg, 1932. 3+508 pp.

This volume concludes the publication of Dedekind's collected works. The high standard of editorial excellence which marked the previous volumes is maintained. There is but one matter for regret: the promised Life by Fricke is not forthcoming, as it apparently had not been written at the time of Fricke's recent death. Instead, however, we have some extremely interesting letters and notes of Dedekind's own.

This volume contains, among other things, the reprints of three classics: items XLVI–XLIX are the famous Supplement XI to Dirichlet's *Zahlentheorie* in its several editions (the last of which are now becoming scarce); item L is *Stetigkeit und irrationale Zahlen*, and item LI is *Was sind und was sollen die Zahlen?* The careful, complete reprints of these great classics alone make the price (M 43.65, bound) of the volume extremely reasonable.

In addition to these there is much more of great historical interest, and much (especially in the theory of groups) that will repay careful study even today. As an historical document giving an insight into Dedekind's manner of facing and overcoming fundamental difficulties, item XLVIII, *Sur la Théorie des Nombres entiers algébriques*, is of particular interest. The account which Dedekind gives of his various inventions may suggest the degree of difficulty his successors may expect in carrying on his work to the next phase, that of the arithmetic (if there is any) of a non-commutative ring. The problems do not seem even to have been clearly formulated yet. To make the basic advances in algebraic numbers which he did, Dedekind was forced to break essentially new ground. A slavish adherence to his methods, which were created specifically for his problem, rather than an emulation of his pioneering spirit, seems unlikely —from present indications—to lead anywhere in particular.

In conclusion, this splendid edition of Dedekind's works will, let us hope, find its way into every mathematical library.

E. T. BELL