SHORTER NOTICES

Principes Géométriques d'Analyse, Deuxième Partie. By Gaston Julia. Paris, Gauthier-Villars, 1932. vii+121 pp.

This book presents a continuation of the first volume of the author dealing with those aspects of the modern theory of functions of a complex variable which are derivable from simple geometrical principles. As the author himself points out in the preface to the first volume, the most important of these principles is the conformal correspondence between two regions of planar character or two Riemann surfaces realized by an analytic function $Z=f(z)$. The first volume is principally devoted to the study of those conformal correspondences which carry a unit surface into a Riemann surface interior to the unit circle. This subject centers about the Lemma of Schwarz and the interpretation of it in terms of the non-euclidean (Lobachevskian) geometry in a circle, indicated for the first time by G. Pick.

The second volume begins with the maximum principle and consists of generalizations of this principle in various directions associated with the names of Lindelöf, Littlewood, Carleman, and others. By a series of easy extensions from the maximum principle the author arrives toward the end of Chapter 1 at the Phragmén-Lindelöf theorem. Some applications are then made to the theory of integral functions. The chapter ends with an important Lemma of Carleman concerning analytic functions $f(z)$ in regions $AOBC$ where $\overline{OA}$ and $\overline{OB}$ are rectilinear segments and ACB is a Jordan arc. The lemma gives a limitation of $|f(z)|$ on the bisector of the angle AOB when the upper bound of $|f(z)|$ on $\overline{OA}$, $\overline{OB}$, and on the arc ACB, and the magnitude of the angle AOB are known.

Chapter 2 is devoted to some classical facts concerning harmonic functions of two variables and the conformal mapping of simply connected regions on a circle. It is rather unfortunate that in connection with the second topic no reference is made to the important work of W. F. Osgood. The author is further led to consider the problem of Dirichlet and in particular the integral of Poisson. One of the methods described for obtaining the integral is that of reducing it to the mean value theorem for harmonic functions. Here again the reference is not wholly adequate since no mention is made of Bôcher, the first to notice the fact. With the mathematical tools thus obtained, the author proceeds to generalize the lemma of Carleman by giving a limitation of $|f(z)|$ valid in every point of the region $AOBC$.

After a brief introduction to the elliptic modular function in Chapter 3, an exposition of the method of Lindelöf follows in Chapter 4. Here the geometric concept underlying the whole work is a relation between two Riemann surfaces S and S_1 which Julia characterizes by saying that S_1 is carried by S. This means that S_1 is a covering surface of S such that every closed curve on S_1 is projected on a closed curve on S. From Lindelöf's fundamental theorem, which contains the lemma of Schwarz as a particular case, a variety of inequalities is developed for functions analytic in a circle. These inequalities, together with some properties of the elliptic modular functions, then yield the classical theorems of Picard, Landau, and Schottky.

In Chapter 5 the author introduces the notion of convexity of functions and derives the fundamental theorems of Hardy and Littlewood concerning the means of order p of analytic functions. In connection with Theorem IV of this chapter, a somewhat analogous theorem of F. Hausdorff may be mentioned. The book ends with a short account of F. Riesz's theory of subharmonic functions and its principal applications to the integral means of analytic functions.

Of the misprints in the book there is one of sufficient importance to warrant mention. In the statement of Lindelöf's fundamental theorem the hypothesis $z_0 = f(x_0)$ (using the author's notation) is omitted.

The book serves the excellent purpose of unifying by means of geometric concepts various branches of the theory of functions which have hitherto been scattered in the literature. The presentation throughout is lucid, rigorous, and elegant.

W. SEIDEL

Richard Dedekind. Gesammelte mathematische Werke. Herausgegeben von Robert Fricke, Emmy Noether, Öystein Ore. Dritter Band. Braunschweig, Vieweg, 1932. 3+508 pp.

This volume concludes the publication of Dedekind's collected works. The high standard of editorial excellence which marked the previous volumes is maintained. There is but one matter for regret: the promised Life by Fricke is not forthcoming, as it apparently had not been written at the time of Fricke's recent death. Instead, however, we have some extremely interesting letters and notes of Dedekind's own.

This volume contains, among other things, the reprints of three classics: items XLVI–XLIX are the famous Supplement XI to Dirichlet's *Zahlentheorie* in its several editions (the last of which are now becoming scarce); item L is *Stetigkeit und irrationale Zahlen*, and item LI is *Was sind und was sollen die Zahlen?* The careful, complete reprints of these great classics alone make the price (M 43.65, bound) of the volume extremely reasonable.

In addition to these there is much more of great historical interest, and much (especially in the theory of groups) that will repay careful study even today. As an historical document giving an insight into Dedekind's manner of facing and overcoming fundamental difficulties, item XLVIII, *Sur la Théorie des Nombres entiers algébriques*, is of particular interest. The account which Dedekind gives of his various inventions may suggest the degree of difficulty his successors may expect in carrying on his work to the next phase, that of the arithmetic (if there is any) of a non-commutative ring. The problems do not seem even to have been clearly formulated yet. To make the basic advances in algebraic numbers which he did, Dedekind was forced to break essentially new ground. A slavish adherence to his methods, which were created specifically for his problem, rather than an emulation of his pioneering spirit, seems unlikely —from present indications—to lead anywhere in particular.

In conclusion, this splendid edition of Dedekind's works will, let us hope, find its way into every mathematical library.

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