ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THIS SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

192. Dr. A. E. Ross: *On criteria for universality of ternary quadratic forms.*

Necessary and sufficient conditions are derived in terms of generic characters that an indefinite ternary quadratic form, classic or non-classic, represent all positive and all negative integers. Such a form must represent zero properly. The theorem for classic ternary quadratic forms was proved by the author (Abstract, this Bulletin, vol. 36 (1930), p. 364) by use of a result of A. Meyer. This paper proves the theorem without use of the equivalence criterion of Meyer and similarly derives the theorem for non-classic forms. Theorem 1. Let f be a properly primitive indefinite classic ternary quadratic form with reciprocal F and determinant D. The necessary and sufficient conditions that f be universal are: D is odd or double an odd integer, $\Omega=\pm 1$, and $(F/p)=(-\Omega/p)$ for every odd prime p dividing $\Delta=D$. Theorem 2. Let f_1 be a primitive indefinite non-classic ternary quadratic form. Consider an improperly primitive form $f=2f_1$ with reciprocal F. Then f_1 is universal if and only if $f=2f_1$ satisfies the following conditions: $\Omega=\pm 1$, the characters of f are $(-1)^{(F-1)/2}=(-1)^{(-r-1)/2}$, $(F/p)=(-\Omega/p)$ for every odd prime p dividing $\Delta=D$, and, in case 4 divides Δ, also $(-1)^{(F^2-1)/2}=1$. These theorems serve as a practical test for determining whether a given ternary quadratic form is or is not universal. (Received July 19, 1932.)

193. Dr. C. F. Luther: *Concerning primitive groups of class u.*

In this paper are proved three general theorems concerning the degree and class of multiply transitive groups. The first gives an upper limit to the degree of a substitution group of class u that contains a substitution of order two and degree $u+\epsilon$ (ϵ a positive integer), and is more than $p_1+p_2+p_3+\cdots p_r$ times transitive, where $p_1, p_2, p_3, \cdots, p_r$ are distinct odd prime numbers and $r>1$. It is $u>n-\epsilon-(n+2\epsilon p_1p_2\cdots p_r)/((p_1-1)(p_2-1)\cdots(p_r-1))$ if ϵ/u is small. Limits are also given for 2, 3, 5, 6, 7, and more than p (a prime)-ply transitive groups. The second theorem gives an upper limit to the degree of a triply transitive group of class $u(>3)$ that contains a substitution of degree $u+\epsilon$ (ϵ a positive integer) and of order p^c (p an odd prime). The third theorem gives an upper limit to the degree of a doubly transitive group of class u that contains a substitution of degree $u+\epsilon$ (ϵ a positive integer) and of prime order p (p an odd prime). (Received July 20, 1932.)

194. Dr. Rothwell Stephens: *Continuous transformations of finite spaces.*

The following problem is considered: Given a finite space P and a knowledge of the existence or denial of eight fundamental properties in P, what possible combinations of these properties can occur in the biunivocal continuous transforms of P. The eight properties considered are those studied by D. McCoy (Tôhoku Mathematical Journal, vol. 33 (1920), pp. 89–116). Solutions are given for several particular types of spaces. In addition several extraneous theorems are proved. For every finite space of type $P4$ of more than one element, the derived set of any set E is given by $K(E)=E+p$. Every finite space of type $P48$ is homogeneous, and for every set E, $K(E)=E$. (Received July 21, 1932.)

195. Professor H. L. Miller: *On the summability of double Fourier series.*

Those writers who have applied Cesàro's method of summing series to the study of the Fourier development of a function of two variables have considered only integral orders of summability. This investigation considers non-integral orders in Cesàro's method of summing the double Fourier series. The Fourier development of a function of two variables, $f(x, y)$, integrable (L) in the region $(-\pi \leqq x \leqq \pi,\ -\pi \leqq y \leqq \pi)$, will be summable (Ck), $k>0$, to the value of the function at any point within the region at which the function is continuous, provided the function remains finite in some cross-neighborhood associated with the point. At a point of discontinuity (x_1, y_1) which lies on a straight line or curve containing all other points of discontinuity in the neighborhood of (x_1, y_1), the development of a function satisfying the same restrictions will be summable (Ck), $k>0$, to a value half way between the limiting values of the function as the point (x_1, y_1) is approached from either side of the line of discontinuity, provided these limiting values exist, except when the tangent to the curve of discontinuity at (x_1, y_1) is parallel to an axis. In this case the series will be restrictedly summable. (Received July 22, 1932.)

196. Professor G. S. Bruton: *Certain aspects of the theory of equations for a pair of matrices.*

Two square matrices A and B with characteristic values α_i and β_i are said to have property P_3 if any polynomial, $f(A, B)$, in A and B has its characteristic values among the numbers $f(\alpha_i, \beta_i)$. Sufficient conditions that A and B have property P_3 are found in terms of common invariant direction. Necessary and sufficient conditions are found for the 1st, 2d, and 3d order cases. Special studies are made where $f(A, B)$ is limited to being $A+B$ or AB. (Received July 22, 1932.)

197. Professor M. H. Ingraham: *A study of certain related pairs of square matrices.*

Dr. G. S. Bruton studied matrices A and B with characteristic values α_i and β_i and such that $f(A, B)$ had characteristics $f(\alpha_i, \beta_i)$. Further necessary conditions that two matrices A and B have this property are found and properties of such pairs are discussed. (Received July 22, 1932.)

198. Professor T. H. Hildebrandt and Dr. I. J. Schoenberg: *On linear functional operations and the moment problem for a finite interval in one or several dimensions.*

In the first part of this paper it is shown that the Riesz theorem that a linear continuous functional operation on continuous functions is expressible in the form $\int_0^1 f d\alpha$, is deducible from and consequently equivalent to the Hausdorff theorem giving necessary and sufficient conditions that a function of bounded variation on a finite interval be determined by its moments $\int_0^1 x^n d\alpha$. This suggests in turn a simple direct proof of the Riesz theorem. The second part of the paper derives results related to Stieltjes integrals in two variables and applies them to the extension of the Hausdorff-Schoenberg results on the moment problem as well as the Riesz theorem in more than one variable. (Received July 22, 1932.)

199. Professor Glenn James: *On Fermat's last theorem.*

This paper considers the Fermat equation $x^n+y^n=z^n$, $z>y>x>0$, for the so called first case and proves that $z-y>c^n n^n$ where c is a certain function of x, y and z whose lower limit is 2. This work provides a simple, and what seems to be a new proof for the case $n=3$, and suggests a point of attack on the general problem. (Received July 23, 1932.)

200. Professor W. A. Manning: *The degree and class of multiply transitive groups*, III.

Let n be the degree and u the class of a group of substitutions G that is neither alternating nor symmetric. The author has stated that if G is 5-ply transitive, $n\leqq 2u$; if 6-ply transitive, $n<5u/3$; if 8-ply transitive, $n<8u/5$; and finally, if G is as much as 12 times transitive, $n<3u/2$. The proofs of the last two limits depend on the proposition that if in a $t(>6)$ times transitive group, all the substitutions of degree $u+\epsilon$ or less are of odd order, then $n<2u-4\epsilon-2t+13$. This is used in combination with the limits found by Dr. C. F. Luther for the degree of t-ply transitive groups of class u in which there are substitutions of degree $u+\epsilon$ of even order. (Received July 23, 1932.)

201. Mr. H. M. Bacon: *An extension of a certain theorem of Kronecker.*

A certain theorem of Kronecker may be stated as follows: Constants $\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_n$ being linearly independent while $\mu_1, \mu_2, \mu_3, \cdots, \mu_n$ are arbitrarily given real numbers, there exists a real number t such that differences $\alpha_j t-\mu_j$, $(j=1, 2, 3, \cdots, n)$, are in absolute value and modulo 1 less than any arbitrarily preassigned number ϵ. This paper shows that if $\epsilon=1/N$ is given, and the constants $\alpha_1, \alpha_2, \alpha_3, \cdots, \alpha_n$ are not linearly independent, then the inequalities $|\alpha_j t-\mu_j|<\epsilon$ (mod 1) may still be satisfied by a real number t provided all relations $c_1\alpha_1+c_2\alpha_2+c_3\alpha_3+\cdots+c_n\alpha_n=0$, with $|c_j|$ less than a certain limit depending upon ϵ, are excluded. It is necessary to exclude all such relations having all $|c_i| \leqq N/(2n)$. It is sufficient to exclude all such relations having at least one of the $|c_i| < h \cdot N$ where $h = (\sqrt{n}/2)(125/48)^{(n^3-n)/12}$. The sufficient condition follows from the application of the Minkowski theory of reduction of quadratic forms to a problem in the geometry of number. (Received July 25, 1932.)

202. Dr. D. H. Lehmer: *A number theoretic machine.*

The number theoretic machine, whose description follows, is designed to handle any problem whose solution is restricted modulo any prime p to approximately $p/2$ cases, as for example in the problem of representing a number by a binary quadratic form using Gauss' method of exclusion. Each prime $p \leqq 113$ is represented by a gear, and each of the possible remainders modulo p corresponds to a hole in the gear. These 30 gears revolve together and ultimately arrive at a position where there is an alignment of holes. This enables a beam of light to pass through all the gears and to fall on a photo-electric cell which automatically stops the machine, so that the solution may be read. The machine sifts 20,000,000 numbers an hour without attention, and runs on the average some 50 hours between solutions. This machine was constructed with funds granted by the Carnegie Institution of Washington. (Received July 25, 1932.)

203. Dr. D. H. Lehmer: *An inversive algorithm.*

Viggo Brun has given an algorithm for calculating the nth prime number from certain values of the function $\pi(x)$, the number of primes $\leqq x$. This paper shows that this algorithm is not peculiar to primes, but on the contrary may be used to exhibit the nth member of any infinite class of positive integers. With this degree of generality it is possible to use the algorithm to obtain identities between numerical functions. (Received July 25, 1932.)

204. Professor Morgan Ward: *The arithmetical theory of linear recurring series.*

This paper considers a sequence of integers defined by a linear recursive relation to any integral modulus m, and reduces the fundamental problem of determining the least period of such a sequence modulo m essentially to determining the period of a mark in a finite Galois field. In the course of the investigation several subordinate problems of arithmetical interest connected with the periods of such sequences are solved. The results are obtained without the use of ideals by transforming the problems into equivalent ones in the theory of congruences to a double modulus which admit of complete theoretical solution. (Received July 25, 1932.)

205. Professor Morgan Ward: *The cancellation law in the theory of congruences to a double modulus.*

This paper shows how to determine all polynomials $U(x)$ such that $A(x)U(x) \equiv 0$ modd m, $F(x)$, where $A(x)$ and $F(x)$ are given polynomials with integral coefficients and m an integer prime to the leading coefficient of $F(x)$. The problem is first reduced to the case when m is a power of a prime p and $F(x)$ is congruent modulo p to a power of an irreducible polynomial modulo p. If $m=p^n$, and p^r is the first elementary divisor corresponding to the prime p of the matrix of the eliminant of $A(x)$ and $F(x)$, then when $n \geqq r$, $U(x)$ must be of the form $p^{n-r}(A_1(x)V_1(x)+pA_2(x)V_2(x)+ \cdots +p^{r-1}A_r(x)V_r(x))$ where the polynomials $A_i(x)$ are known and the polynomials $V_i(x)$ are arbitrary. The result when $n \leqq r$ is similar. (Received July 25, 1932.)

206. Professor Morgan Ward: *A type of abstract arithmetic allied to a Boolean algebra.*

This paper investigates an abstract associative, commutative, and idempotent operation, of which finding the L.C.M. and G.C.D. of two numbers are instances, with a view to ascertaining under what conditions the elements of a set closed with respect to such an operation may be uniquely resolved into prime factors in the sense of elementary arithmetic. The results may best be summarized by thinking of the elements of the set as whole numbers and the operation as that of finding the L.C.M. If it is postulated that every element has only a finite number of non-equivalent divisors, the G.C.D. operation can be defined over the system in terms of the L.C.M. If in addition it is assumed that the L.C.M. operation is distributive with respect to the G.C.D., unique factorization is obtained. The system thus obtained is contrasted with a Boolean algebra and shown to form a distinct species of non-numerical algebra. (Received July 25, 1932.)

207. Dr. E. J. Purcell: *Involutorial space Cremona transformations determined by non-linear null reciprocities.*

Birational correspondences of space are treated, for the greater part analytically, in which any two corresponding points are reciprocal with respect to a quadric polarity, or in a null-system. This paper is particularly concerned with the involutorial cases. (Received July 25, 1932.)

208. Dr. A. E. Ross: *On representation of integers by ternary forms.*

This paper considers representation of integers by ternary quadratic forms. A very simple relation is established between the generic characters of such a form f and the arithmetical progressions of integers (not necessarily prime to the determinant D of f) not represented by f. The ternary forms of different genera cannot represent the same totality of integers. In the case of indefinite ternary forms, Meyer's (A. Meyer, Journal für Mathematik, vol. 108 (1891), pp. 125–39) criterion for equivalence is employed to obtain general theorems on representation. In studying ternary forms a method is introduced which reduces the problem of representation of integers by a general positive or indefinite ternary quadratic form of invariants Ω, Δ to that of representation of multiples of appropriately chosen primes p, q by a form $2^kpx^2+\Omega y^2+\Omega\Delta 2^l qz^2$, where k and l each are equal to either 0 or 1. (The method has been generalized to apply to forms in n variables.) In the case of positive forms this latter method permits a similar reduction of the problem of determination of the number of representations. For example $N(x^2+2y^2+2z^2-2yz=m)=N(x^2+2y^2+3z^2=2m)$. (Received July 25, 1932.)

209. Miss Alta Odoms: *On the summability of triple Fourier series.*

This paper applies the Cesàro method of summation, particularly for positive non-integral orders, to the Fourier development of a function of three variables, $f(x, y, z)$. The functions considered are integrable in the sense of Lebesgue and are finite in a cross region associated with the point under consideration. It is shown that the Fourier development of such a function is summable (Cr), for any value of $r>0$, at every point of continuity of the function interior to the region $(-\pi\leqq x\leqq\pi,\ -\pi\leqq y\leqq\pi,\ -\pi\leqq z\leqq\pi)$ to the value of the function at that point. It is also shown that, if (x_1, y_1, z_1) is a point of discontinuity of $f(x, y, z)$ such that every other point of discontinuity in its neighborhood lies on a plane through (x_1, y_1, z_1) or on a curved surface through (x_1, y_1, z_1) whose tangent plane at (x_1, y_1, z_1) is not parallel to any of the coordinate axes, and if $f(x, y, z)$ approaches a definite value as (x_1, y_1, z_1) is approached from either side of the surface, the Fourier development of $f(x, y, z)$ is summable (Cr), for any value of $r>0$, at (x_1, y_1, z_1) to a value half-way between the limiting values of the function at that point. Other cases of discontinuity discussed offer more complicated results. (Received July 25, 1932.)

210. Dr. Max Coral: *On extremal surfaces.*

New proofs are given for the conditions of Haar-Lagrange, Legendre, and Weierstrass which are necessary in order that a surface $z=Z(x, y)$ of class C' in a region B may minimize an integral $I=\int\int_B f(x, y, z, p, q)dxdy$ in a given class of surfaces with the same boundary. The properties of the minimizing surface are studied in a rectangular neighborhood $x_1\leqq x\leqq x_2$, $y_1\leqq y\leqq y_2$, of an interior point of the surface by substituting $z=Z(x, y)+X(x)Y(y)$ in the integral I, X and Y being functions of class C' which vanish at the ends of their respective intervals, and carrying out the integration as to y. The resulting integral $J=\int_{x_1}^{x_2} g(x, X, dX/dx)dx$ has a minimum for $X(x)\equiv 0$ in the class of functions $X(x)$ of the above described type, and the well known necessary conditions for this simple variation-problem may be interpreted for the original two-dimensional problem. The method extends to variation-problems with integrals of any multiplicity and containing any number of dependent variables. (Received July 25, 1932.)

211. Professor B. A. Bernstein: *Remarks on propositions* *1.1 *and* *3.35 *of Principia Mathematica.*

The writer's aim is to show that the interpretation of proposition *3.35 in Whitehead and Russell's *Principia Mathematica* is inadmissible; to show that the *Principia's* account of the nature of *1.1 and its relation to *3.35 is inadmissible; and to make clear the relation to *1.1 of his own proposition 1.1, given by him previously (this Bulletin, vol. 37, pp. 480–488) as the Boolean form of *1.1. (Received July 25, 1932.)

212. Professor A. A. Shaw: *The place of Ananiah Shiragooni in the history of mathematics.*

This is a continuation of the author's paper read before the November meeting of 1931, where Ananiah Shiragooni was quoted as the only source of reference available. Judging from the Jerusalem MS, Shiragooni's mathematical work is divided into three headings: (a) Concerning Weights and Measures, where he gives eighteen tables, six of which give both multiples and submultiples of the measures in question. (b) Concerning Measures of Capacity, with nineteen tables. In both classes of tables Shiragooni gives equivalent names in Hebrew, Greek, Latin, Armenian, and Egyptian with occasional etymology. These tables are in general accurate. The author found only three errors, which may be scribal. (c) Arithmetical problems, forty in number, usually very long and descriptive and referring to war, hunting, fishing, commerce, church, wine, fruits, fountain problems, etc. Only the answers are given in Armenian alphabetic numerals, as throughout the tables. No hint is given regarding the method of operations in these numerals. Probably simple operations were performed in those symbols and complicated ones by use of the abacus. Comparing the Jerusalem and the Venice MSS of Shiragooni, the author finds interesting variant forms of numerals in corresponding tables. (Received July 25, 1932.)

213. Professor Raymond Garver: *Concerning the limits of a measure of skewness.*

In a note in the Annals of Mathematical Statistics (vol. 3 (1932), pp. 141–142), Hotelling and Solomons have proved that the measure of skewness (mean —median)/standard deviation must lie between -1 and 1. In this note another proof is given; its interest lies in the fact that the theorem is shown to be essentially a generalization of a well known algebraic inequality. (Received July 26, 1932.)

214. Professor Raymond Garver: *The Edgeworth taxation phenomenon.*

In a number of papers Edgeworth has considered an interesting problem in mathematical economics. A monopolist is offering two rival commodities in the same market; a tax is imposed on the more expensive of the two. Edgeworth wished to show that it might then be (though admittedly it would not ordinarily be) to the advantage of the monopolist to *lower* both prices. In this paper the demand functions used by Edgeworth are criticized; certain of their economic implications seem to render them unsuitable. Another example is then developed, in which simpler demand functions are used, but which requires more complicated expressions for cost of production than those employed by Edgeworth. (Received July 26, 1932.)

215. Professor Raymond Garver: *Remarks on continued fractions.*

(1) Limits for the accuracy of the nth convergent in the simple continued fraction expansion of the square root of an integer are found which are usually

considerably better than the limits ordinarily given. (2) Two methods for approximating square roots are analyzed by comparing them with the continued fraction method. Some results of this kind were given by Günther in 1882. (Received July 26, 1932.)

216. Dr. I. J. Schoenberg: *On finite-rowed systems of linear inequalities in infinitely many variables*, II.

In a previous paper with the same title (Transactions of this Society, July 1932) a certain class of systems of linear inequalities in infinitely many variables was solved and applications to the theory of completely monotonic functions were derived from a particular type of such systems which were called Hausdorff systems. The present paper gives an extension of these results to a similar class of systems of linear inequalities involving a double sequence of variables. As an application of these results, the theorems of F. Hausdorff, S. Bernstein, and D. V. Widder concerning completely monotonic functions of one variable are extended to such functions of two independent variables. In particular, explicit expressions of such functions in any finite or infinite rectangular region are obtained. This paper will appear in the Transactions of this Society. (Received July 26, 1932.)

217. Professor G. T. Whyburn: *Note on a homogeneity property.*

A continuum M will be said to be arc-homogeneous provided that if s and t are any two simple arcs lying in M then there exists a topological transformation of M into itself throwing s into t. In this note it is shown that every compact locally connected and arc-homogeneous continuum of dimension 1 is a simple closed curve. A similar conclusion may be obtained even under a weaker and localized arc-homogeneity condition. (Received July 26, 1932.)

218. Professor H. S. Wall: *On continued fractions in which the coefficients have limiting values.*

Let (1) $1+[b_n z/1]_1^\infty$ be a continued fraction in which $b_n \neq 0$, $\lim_{n=\infty} b_{kn+i}=L_i$, $i=1, 2, \cdots, k$. Let α_n/β_n be the nth convergent of $1+[L_n z/1]_1^\infty$ $(L_{i+mk}=L_i)$, and set $Z(z)=(-1)^{k-1}z^k L_1 L_2 \cdots L_k/(\beta_k-\beta_{k-1}+\alpha_{k-1})$.[2] Then (1) converges over every bounded closed region containing no point z_0 which is a pole of Z, or such that $Z(z_0)$ is real and $\leqq -1/4$, with the exception of certain isolated points which are poles of the otherwise analytic limit. For $k=1, 2$ this gives two theorems of E. B. Van Vleck. A sufficient condition proved by Van Vleck that (1) shall be a meromorphic function is (2) $\lim b_n=0$. He showed the necessity of the condition when $b_{2n}b_{2n+1}>0$, b_n real. If some but not all the L_i are 0, and $\beta_k-\beta_{k-1}+\alpha_{k-1}\equiv 1$, (1) is meromorphic yet (2) fails. Another sufficient condition is found which requires $\lim b_{2n}q_n=\lim (b_{2n+1}/q_n)=0$, q_n a certain function of $b_1, b_2, \cdots$ and a parameter α, which reduces to 1 if $\alpha=0$. (Received July 26, 1932.)

219. Professor A. D. Michal and Dr. R. S. Martin: *Invariant functionals of functional forms.*

This paper continues the investigations of Professor Michal (A. D. Michal, American Journal of Mathematics, 1928) on functional invariants of functional forms. A number of uniqueness theorems involving Stieltjes integrals are proved. The paper includes theorems on groups of transformations that leave functional forms invariant. (Received July 26, 1932.)

220. Professor A. D. Michal and Mr. J. L. Botsford: *Simultaneous differential invariants of an affine connection and a general linear connection.*

This paper deals with invariants depending on a general König linear connection and its derivatives as well as on a symmetric affine connection and its derivatives. The properties of "geodesic representations of order r" are deduced and applied to the derivation of König normal tensors and König extensions of composite tensors. These developments are used to prove a replacement theorem for the simultaneous tensor differential invariants of the aforementioned König connection and affine connection. This theorem includes as a special case the Thomas-Michal replacement theorem for differential invariants of a symmetric affine connection (Annals of Mathematics, vol. 28 (1927), pp. 196–236; A. D. Michal, this Bulletin, vol. 36 (1930), pp. 541–546). (Received July 26, 1932.)

221. Mr. A. H. Clifford: *Geometry of a normed abelian group manifold.*

Collineations and motions in a normed Abelian group manifold are considered in this paper. Several theorems on groups of collineations and motions are proved. Specializations of the above results give interesting theorems in various functional geometries. (Received July 26, 1932.)

222. Dr. W. Cauer: *The Poisson integral for functions with positive real part.*

A representation is given for a function analytic in a half-plane and having non-negative real part there. This representation is analogous to and derived from the Poisson-Stieltjes integral, well known in the case where the function is analytic in the unit circle. The paper will appear in full in an issue of this Bulletin. (Received July 28, 1932.)

223. Professor G. A. Bliss and Dr. I. J. Schoenberg: *On the derivation of necessary conditions for the problem of Bolza.*

In this paper it is shown that the methods used by Bliss to derive the first necessary conditions for the problem of Lagrange with variable end-points (American Journal of Mathematics, vol. 52, pp. 690–693) can be adapted with very slight modifications to deduce analogous theorems for the problem of Bolza (see Bliss, Annals of Mathematics, (2), vol. 33, pp. 261–274). It is further shown how, by the introduction of some new variables, the problem of Bolza in the parametric form considered by Morse and Myers (Proceedings of the American Academy of Arts and Sciences, vol. 66, pp. 235–253) can be immediately reduced to one of the Bolza type. The necessary conditions indicated by Morse and Myers, as well as their criteria concerning normality, then turn out to be simple corollaries of the corresponding theorems for the problem of Bolza. For the problem of Mayer as formulated by Myers (this Bulletin, vol. 38, pp. 303–312) a similar method is applicable. (Received July 28, 1932.)

224. Professor James McGiffert: *Particular solutions of* $y''+c_1x^py'+c_2x^{p-1}y=0$, *which consist of the product of a polynomial by an exponential.*

By application of infinite series, one obtains the following solution, $y=[x^{k-p}-((1/c_1)(k-p)(k-p-1)/(p+1))x^{k-2p-1}+(1/c_1)^2\{(k-p)(k-p-1)\cdot(k-2p-1)(k-2p-2)/((p+1)(2p+2))\}x^{k-3p-2}+\cdots]\,e^{\mu}$, where μ is equal to $-c_1x^{p+1}/(p+1)$, and where $k=c_2/c_1$. The expression in the brackets, multiplying the exponential portion of this solution, will be a polynomial in x for all positive integral values of p, and for all values of k such that $k=I(p+1)$ or $k+1=I(p+1)$, where I is a positive integer. For all other values of k, the expression in brackets will be found to be an infinite series, and the solution can then not be of finite form. (Received July 30, 1932.)

225. Dr. Charles Wexler: *On real quadratic fields.*

It is not unusual, in computing the fundamental units of real quadratic fields by the classical method, to find that the coefficients of the unit are numbers of fifteen or more digits. It is then difficult to check the accuracy of the computation in the ordinary way, which involves the squaring of the coefficients. In this paper are proved theorems that, among other things, simplify the verification of the coefficients. (Received August 3, 1932.)

226. Professor Orrin Frink: *Jordan measure and Riemann integration.*

It is shown in this paper that the Riemann integral can be defined in terms of Jordan measure. The situation is slightly more complicated than in the case of the Lebesgue integral; for example, the set of points at which a function integrable Riemann is $>k$ is not necessarily measurable Jordan. A necessary and sufficient condition for Riemann integrability is that for all except a countable number of values of k the "Lebesgue" sets at which the function $>k$, and $<k$, are measurable Jordan. Many other similar necessary and sufficient conditions are given. As one consequence it is shown that a function integrable Riemann can be uniformly approximated by functions which take on only a finite number of values, at sets measurable Jordan. (Received August 25, 1932.)

227. Mr. Walter Leighton: *On the convergence of continued fractions in which the elements are independent variables.* Preliminary report.

Let A_n/B_n be the nth convergent of (1) $x_0+[1/x_n]_1^\infty$, a continued fraction in which $x_0, x_1, \cdots$ are arbitrary complex variables, and set $g_n=\sum_{i=0}^n x_{2i}$. Then if the two series $\sum[x_{2n}/(g_{n-1}g_n)]$ and $\sum[x_{2n-1}g_{n-1}^2]$ converge absolutely, there exist quantities α, β, α_1, β_1, to which A_{2n}/g_n, B_{2n}/g_n, A_{2n+1}, B_{2n+1}, respectively converge for $n=\infty$; and which satisfy a relation of the form $\alpha\beta_1-\alpha_1\beta=\delta$. Here δ is a finite quantity which is zero if and only if $\lim_{n=\infty}|g|=\infty$, and in this case (1) converges (at least in the wider sense). Seven other convergence criteria of which this is an example are also obtained. (Received August 10, 1932.)

228. Dr. E. W. Miller: *Solution of the Zarankiewicz problem.*

The following question has been raised (Fundamenta Mathematicae, vol. 7 (1925), p. 381, problem 37) by C. Zarankiewicz: Is every acyclic continuous curve homeomorphic with some proper subset of itself? It is the purpose of this paper to show that the above question is to be answered in the negative. An acyclic continuous curve S is constructed which contains a set κ with the following two properties: (1) Each point of K is a fixed point with respect to any (1, 1) bicontinuous transformation of S into a subset of itself. (2) Each point of S of (Urysohn-Menger) order >1 lies on an arc of S whose end points are points of K. It is easily shown that such an acyclic continuous curve can be homeomorphic with no proper subset of itself. (Received August 29, 1932.)

229. Dr. W. M. Rust: *A theorem on Volterra integral equations of the second kind with singular kernels.*

The Volterra integral equation of the second kind with kernel $K(x, y)$ which satisfies the following two conditions: (i) if $r(x)$ is bounded and continuous so is $\int_0^x K(x, y)r(y)dy$, (ii) for n sufficiently great the iterated kernel $K_{2^n}(x, y)$ is bounded and continuous, has a unique, bounded, continuous solution which is the solution of the equation (a) $u_n(x)=\sum_{m=1}^{2n-1}\int_0^x K_m(x, y)f(y)dy+\int_0^x K_{2^n}(x, y)u_n(y)dy$, $f(x)$ being bounded and continuous. The proof, which is elementary, consists in showing that a bounded and continuous solution of the equation of the type (a) involving only kernels up to $K_{2^i}(x, y)$ is also a solution of the corresponding equation involving only kernels up to $K_{2^i-2}(x, y)$. (Received August 19, 1932.)

230. Professor Norbert Wiener and Dr. R. C. Young: *The total variation of* $g(x+h)-g(x)$.

A new proof is given of Plessner's theorem, that $f(x)$ is absolutely continuous when and only when the total variation of $f(x+h)-f(x)$ tends to 0 with h. The set of values of h for which the total variation of $f(x+h)-f(x)$ is not the sum of the total variations of $f(x)$ and $f(x+h)$ is null if $f(x)$ is constant in the complementary intervals of a closed null set. The null set of values of h in question may be denumerable, finite (including non-existent), or of the order of the continuum. Examples of all three possibilities are given, the denumerable case being due to Hille and Tamarkin. (Received August 2, 1932.)