Theory of Functionals and of Integral and Integro-differential Equations. By Vito Volterra. Edited by Luigi Fantappiè. Authorized Translation by Miss M. Long. London and Glasgow, Blackie & Son, Ltd., 1930.

The book at hand is at once a translation into English and a revision and extension of a series of six lectures delivered by Volterra at the University of Madrid in 1925. Its expressed purpose is to develop in the reader an interest in and some familiarity with the theory of functionals. In this the author will in all likelihood be found highly successful.

By way of definition, z is a functional of the function $x(t)$, if under a specifiable law there corresponds one and only one value of z to each and every function $x(t)$ defined on a given interval $a \leqq t \leqq b$. The relation

$$z = \int_a^b k(t)x(t)dt,$$

in which the function $k(t)$ is specified, may be regarded as a simple example.

Beginning with definitions and generalities, the author develops his theme rapidly and carries the discussion over the problems of the functional calculus, the theory of composition and of permutable functions, functionals as a generalization of analytic functions, the theories of integral and integro-differential equations, and a survey of the extensive field of applications.

The presentation retains much of its original character as a series of lectures and is, therefore, a survey, not a treatise. The amount of detail is cut to a minimum, and is adjusted (a mark of notable expository skill) to suffice in each case for the clear presentation of the problem at issue and the method of its resolution, without, however, obscuring the general outlines on which the reasoning is planned.

For the reader versed in the general processes of analysis this book is to be recommended without reservation. An extensive bibliography covering publications to 1929 is included at the end of each lecture.

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Algebra. By Oskar Perron. Second revised edition. Vol. I, *Die Grundlagen.* Berlin and Leipzig, Walter de Gruyter, 1932. vi+301 pp. 12.80 Mk.

As the first edition of this capable general book on the fundamentals of algebra was reviewed in this Bulletin (vol. 34, p. 115), we shall here speak only of the changes that have been made in this new edition—and of changes that the reviewer wishes had been made. Taken as a whole, the new one is about the same as the old. If anyone familiar with the first edition looks at the revision, he certainly gets the impression that the latter is scarcely more than a reprint with a few secondary editorial changes.

In the third chapter, the author corrects the definition of the product of two determinants of order n from that in which we use rows of the first with rows of the second to that in which we use rows of the first with columns of the second. He still, however, makes the mistake of defining the product of two matrices (both with k rows and l columns) by using rows of the first with rows of the second, although he saves the situation by never applying this definition to the product of three or more (where it is well known that the associative law does not hold) and by so formulating the proofs of theorems that he needs the notion of the product of two matrices precisely in this form. It is, however, regrettable that he does not give the orthodox definition of the product of two matrices (rows of the first with columns of the second), since this is the only one of the four natural possibilities (row or column of first with row or column of second) that admits the associative law and hence is the one that has to be used in applications of matrix-theory to linear transformations, group-matrices and the like. Moreover, it seems very peculiar in this day and age to omit the notion of a matrix as a hypercomplex quantity in any general book on algebra.

To this same chapter he adds several pages on the elements of linear homogeneous transformations, and these are good as far as they go, but they do not go far enough.

In his fine fifth chapter on resultants of two or more polynomials in any number of variables, he makes a few minor changes in his rather long series of theorems (overwhelming to the average beginning graduate student); but, although this is, up to the present, the best treatment of the subject by elementary methods (that is, not using Kronecker's theory of modular systems), yet one can not suppress a wish that some one would evolve a neat, adequate treatment that does not involve such a multitude of details. Moreover, use of both editions as reference books in a general graduate course on algebra makes one a little irritated at the author's apparent inability to see that Theorem 117 (§43), Theorem 118 (§44), Part II of §34, and Theorem 134 (§52) can be proved easily as corollaries of the fact (Theorem 115) that the resultant of two polynomials f and g in one variable is a polynomial in the coefficients of f and g whose vanishing is a necessary and sufficient condition that f and g have a common factor and the fact that the discriminant of a polynomial f in one variable is a polynomial in the coefficients of f whose vanishing is a necessary and sufficient condition that f have a repeated factor. Similarly, for Theorem 124 (=old Theorem 126) he uses the long-winded proof of the first edition instead of noting that this theorem follows immediately from the last equation before the statement of the theorem.

The proof of the fundamental theorem of algebra has been changed in a vital way, for instead of giving a variation of Walecki's proof, he gives the proof due to Artin, Schreier, and Dörge (Hamburg Seminar, 1927, and Sitzungsberichte der Berliner Akademie, 1928). In the application of resultants to conditions for solvability of algebraic equations in any number of variables the author adds two significant theorems (135 and 136) and the treatment of a system of resultants of a system of l non-constant polynomials in $k+l$ variables. He also adds a section (56) on the application of resultants to questions as to the factorability of a polynomial, and a section on properties of the resultant and the multiplicity of a solution.

In other words, this book remains a fine text-book in the old-fashioned highly respectable algebra, although the reviewer could not restrain a feeling of regret that the author gives no hint of the fascinating newer point of view and theory that have come into such prominence during the last five years.

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