

submitted to the editors of the *Mathematische Annalen* shortly before the World War. It contained numerous points of contact with the results of Chapter VI (Dependence of the characteristic values of elliptic boundary value problems on the domain and on the coefficients of the problem, zeros of the fundamental functions and the like). A reference to a recent paper by S. A. Janczewsky (*Annals of Mathematics*, vol. 29 (1928), pp. 521–542, particularly p. 542) would not be out of place on page 393 in connection with the corrected statement of a property of the nodes of the fundamental functions. It was Janczewsky who pointed out that the corresponding statement on page 365 of the first edition was not correct.

Some of the mistakes of the first edition are corrected in the present one, while some others are not. It is a little disappointing, for instance, that the erroneous treatment of zeroes of Bessel's functions (pp. 427–428) and an inconsistent definition of the Legendre's functions of the second kind (p. 436), which were pointed out by the reviewer of the first edition, are preserved in the second edition. A certain elegant vagueness of the style, which was also noticed by the reviewer of the first edition, of course, is preserved in the second one. It is desirable, however, that the statements of some results be made more precise. For instance, the statement of Picard's necessary and sufficient conditions for the existence of a solution of a Fredholm integral equation of the first kind is not correct as it stands on pages 135–136. The boundedness of the integrals  $\int K(s, t)^2 ds$ ,  $\int K(s, t)^2 dt$  on page 129 is not needed for the possibility of extension of the classical results of Fredholm theory to unbounded kernels. The statement concerning the sequences converging in mean (p. 93), which was erroneous in the first edition (pp. 96–97), is now correct, but fails to emphasize the essential fact that the function  $f(x)$  to which the sequence converges in mean is uniquely determined, and that the whole property is merely a modification of Cauchy's criterion for the existence of a limit of a sequence, and states in essence the completeness of Hilbert's space.

The present reviewer shares with his predecessor the eager expectation (not yet realized) of the promised Volume II of the treatise. A quantity of most important facts concerning the existence of solutions and the precise formulation of the conditions under which these solutions exist are referred to the second volume. Without this volume at hand, the reader can not help the feeling of a certain lack of a solid foundation for many theories developed in the first volume. Hence, thankful as we are for the completion of the present second edition of the first volume of this excellent treatise, we have to agree with one of the authors of the book (R. Courant): "Das Erscheinen dieser zweiten Auflage legt mir mit verdoppelter Stärke die Verpflichtung auf, den zweiten Band, mit dem zusammen dieser vorliegende erst ein abgeschlossenes Ganzes bilden wird, in Druck zu geben."

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*Algebraische Theorie der Körper*. By Ernst Steinitz. Edited by Reinhold Baer and Helmut Hasse. Berlin and Leipzig, de Gruyter, 1930. 150+27 pp.

The present book is a separate edition of Steinitz' well known paper on abstract fields, which appeared in the *Journal für Mathematik* (vol. 137 (1910)). Steinitz's paper has given rise to numerous investigations on abstract fields and

rings and it will always be considered as one of the classics of this branch of mathematics. I shall not give any account of the contents of this paper since I have already in a recent address (this Bulletin, August, 1931, discussed Steinitz's results and some of their consequences.

It is of interest to observe that Steinitz had but few precursors in these investigations, in spite of the fundamental character of his paper; only the ideas of Dedekind from the supplements to the fourth edition of Dirichlet-Dedekind's *Number Theory* have a direct bearing on Steinitz' problem, and they must have been particularly useful in the investigations on adjunctions of finite rank. The problem to determine all fields in which the Galois theory of equations is valid seems first to have been proposed by H. Weber.

Steinitz's paper is for the greater part easily read; only the chapter on Galois fields, particularly where the Zermelo axiom is used, seems unnecessarily complicated. To remedy this the two editors have added an appendix of explanations and notes and some of the most complicated proofs have been simplified. For those students who want to use the book as an introduction to abstract algebra these notes will be a valuable assistance; if I should venture a criticism it might be that they are in a few places a little too obvious. Very good presentations of the ideas of Steinitz can also be found in Haupt's *Algebra* (Leipzig, 1929) and in the first volume of van der Waerden's *Moderne Algebra* (Berlin, 1930).

Although the paper by Steinitz clearly aims at a discussion of the general Galois theory, this part has not been completed by Steinitz. In a second appendix the editors give a short representation of the general theory both for finite and infinite fields. The new edition also contains an index of terms and a picture of Steinitz.

I should like to recommend the book to students of algebra; for teachers of advanced algebra it would make a very suitable nucleus for a short seminar on abstract fields.

Finally, I shall make an observation without great consequence regarding a remark in the introduction, namely, "The appendix and explanations are mainly due to the younger of the two editors." It seems that this statement does not have the decisiveness expected in mathematics, as it cannot be supposed that all readers have made the personal acquaintance of both editors. I know that both editors are young men, and it seems necessary that the exact age should be stated.

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