Cours de Mathématiques Générales (Analyse et Géométrie). By R. Garnier. Vol. 2. *Calcul Intégral*. Paris, Gauthier-Villars, 1931. 6+396 pp.

The first chapter of this volume (Chapter 12 of the two-volume course) is devoted to the exposition of integration of continuous functions. The idea of definite integral is introduced before that of indefinite integral. Chapter 13 deals with double integrals and triple integrals. Chapter 14 takes up line integrals and surface integrals in the customary manner, with some vector analysis.

Chapter 15 is on ordinary differential equations. The discussion is elementary, yet more complete than is usual. Among other topics we find: singular integrals, limiting curves, and the delimitation of regions of a plane covered by integral curves associated with different analytic forms of integrals.

Chapter 16 is on linear partial differential equations. The geometric discussion of first-order equations is quite clear. The higher order equations are illustrated by the equation of the vibrating cord, with the d'Alembert and Bernoulli solutions. Several examples of Fourier series terminate the chapter.

There are six notes at the end of the volume: oblique coordinates, systems of sliding vectors, envelopes of straight lines in normal form, practical decomposition of rational functions with complex poles, explicit integration of certain indefinite integrals by means of inverse circular or hyperbolic functions, the correspondence between circular and hyperbolic functions.

There are three obvious misprints: page 13, line 25; page 38, line 2, and page 208, line 3. The author is to be commended for his insistence upon the precautions to be taken in changing variables in integration, especially in the choice of proper determinations of inverse functions. The treatment of systems of ordinary linear differential equations with constant coefficients might have been more complete. The material of Chapter 15 (110 pages) supplemented by a goodly number of problems would make an interesting course for American undergraduates: the other chapters are covered, in general, in many of our advanced calculus texts.

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Darstellung und Begründung einiger neuerer Ergebnisse der Funktionentheorie. By Edmund Landau. Second Edition. Berlin, Julius Springer, 1929. 122 pp.

This new edition of Landau's well known work is a welcome contribution to modern literature on the theory of functions. It is by no means a mere reprint of the first edition (1916). Some new material has been added, some proofs simplified, and in fact, as Landau points out, one fourth of all the references are to papers which have appeared since the publication of the first edition.

The book treats a number of selected topics in the theory of functions of a complex variable, particularly topics which either in subject-matter or method are concerned directly with power series, topics to which Landau himself contributes definitely in discovery or presentation. The book does not claim to include all the principal results of modern function theory; for instance the names of Jensen, Julia, Montel, and Ostrowski do not appear.

Chapter I is concerned with limited analytic functions, and has been enriched since the first edition by the addition of Carathéodory's proof of Fatou's theorem, namely that a function analytic and limited in the unit circle C possesses boundary values for radial approach to the circumference almost everywhere on C. This is a useful addition, for Fatou's theorem is of great importance, for instance, in proving the validity of Cauchy's integral formula for the function or in the study of conformal mapping, and Carathéodory's proof is entirely elementary, not requiring even the formal use of Lebesgue integration.

Chapter II, on summability, is devoted chiefly to a proof of the Knopp-Schnee theorem of the equivalence of the methods of Cesàro and Hölder. Chapter III is on the converse of Abel's theorem relative to the continuity of the function represented by a power series, the main result being due to Hardy and Littlewood. Chapter IV contains some specific examples to illustrate various possibilities as to the nature of the convergence of a power series on the circle of convergence.

Chapter V studies the connection between the coefficients of a power series and the singularities of the function represented; it contains particularly M. Riesz's theorem that if $a_n \to 0$, the power series $\sum a_n x^n$ converges in each point of the unit circle of regularity of the function represented, and the convergence is uniform on any closed arc of regularity of the function; and Fabry's theorem on singular points.

Chapter VI is on the maximum and minimum values of the modulus of an analytic function. Hadamard's three-circle theorem is proved and applied to the proof of Jentzsch's theorem. The proof by Pólya and Szegö of Hardy's theorem on the monotonic character of

$$I(r) = \int_0^{2\pi} \left| f(re^{i\phi}) \right| d\phi$$

is also given. It would have been worth while to have given the corresponding result (perhaps also as proved by Pólya and Szegö) for the more general function

$$I_p(r) = \int_0^{2\pi} \left| f(re^{i\phi}) \right|^p d\phi, \qquad (p > 0).$$

Chapter VII treats Picard's theorem on essential singularities, together with the related theorems of Bloch, Landau, and Schottky. Chapter VIII is the only one dealing with conformal mapping as such, although Chapter III gives Fejér's theorem on the convergence of the power series for a smooth (schlicht) function, and gives particularly the proof of Koebe's theorem on distortion.

The entire work is written in Landau's characteristic style—concise, clear, complete. The proofs are simple and are carried through in full detail, and the formulation, correlation, and presentation of such proofs represents the most important service rendered by the book. The reviewer could wish that the supplementary remarks were more elaborate at times, that motivation for some of the proofs was given, and that there were here and there hints of elaboration and application of results, carried out in detail elsewhere in the literature. Such additions would make the book more valuable for the beginner. But the book is already valuable for the beginner, and is indispensable for the more advanced student.

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