Introduction to the Theory of Fourier's Series and Integrals. By H. S. Carslaw. Third edition, revised and enlarged. London, Macmillan and Company, 1930. xiii+368 pp.

Fouriersche Reihen. By Werner Rogosinski. (Sammlung Göschen, No. 1022.) Berlin and Leipzig, de Gruyter, 1930. 135 pp.

The second edition of Professor Carslaw's book was reviewed in vol. 28 (1922) of this Bulletin. The third edition, while substantially of the same scope as the second, is in no sense a mere reprinting of the latter edition. There has been considerable rewriting throughout the book and a number of noteworthy additions. The revisions are all advantageous and serve to bring the work into closer harmony with the present status of the theory of Fourier series.

The most important addition is an appendix on Lebesgue's theory of integration, with a few of the applications of this theory to the discussion of Fourier series. In a combined review of the second edition of Carslaw's book (see reference above) and volume I of the second edition of Hobson's *The Theory of Functions of a Real Variable and the Theory of Fourier's Series* the present reviewer expressed the opinion that sooner or later certain workers in the field of applied mathematics might well find their center of interest transferred from the Riemann integral to the Lebesgue integral. The appearance of a treatment of the Lebesgue theory in a book written primarily for those interested in the applications of Fourier series seems to prove that the drift of the times is in line with the reviewer's prediction. There is a certain neatness and elegance in the theory of Fourier series, as based on the Lebesgue definition of the integral, which is lacking in any discussion that is limited to the Riemann definition. This leads to the inevitable conclusion that the Lebesgue integral is the natural tool in dealing with Fourier series and that it must eventually be introduced even in books on the applications. It seems to the reviewer that it is only a question of time until the elements of the Lebesgue theory filter down into such courses as advanced calculus, and then there will be no valid reason for avoiding it in books on Fourier series.

The treatment of the Lebesgue integral in Appendix II of Carslaw's book is an excellent brief discussion along the lines of the expository treatments due to de la Vallée Poussin and is a valuable addition to the available literature on this fundamental topic. The other additions to the book that should be mentioned are: an extension and revision of the historical introduction to take fuller account of recent literature; more complete discussion of monotonic sequences, upper and lower limits of indetermination, second law of the mean, uniform convergence, and term by term integration; and finally the inclusion of the Heine-Borel theorem, Parseval's theorem, and a discussion of functions of bounded variation.

Professor Rogosinski's book deals exclusively with the theory of Fourier series and presupposes familiarity with various topics from the general theory of functions of a real variable treated by Carslaw. Therefore, although the general policy with regard to the series in which the book appears has restricted the author to a volume of much smaller extent, he has succeeded in discussing virtually all the topics directly connected with Fourier series that are treated in Carslaw's book, though not always in as much detail. In addition he touches on certain additional topics, such as the uniqueness of the development. Like

Carslaw he bases his treatment of the subject on the Riemann integral, though referring occasionally to generalizations that only have significance in the domain of the Lebesgue theory.

Between the table of contents and the text there is given a brief list of books that go more deeply into the theory of Fourier series than the present work. This list furnishes a curious instance of the failure of some continental writers to keep up to date with regard to works published in English. It contains a reference to the first edition of Hobson's *The Theory of Functions of a Real Variable and the Theory of Fourier's Series* (which dates back to 1907), but makes no mention of the second edition in two volumes (vol. I (1921), vol. II (1926)), or the third edition of volume I (1927), although continental works of later date are cited. In view of the fact that volume II of the second edition of Hobson's work contains the most extensive account of existing literature on Fourier series available in any language, such an omission is difficult to explain.

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Geometrische Transformationen. By Karl Doehlemann. Second edition prepare., by Wilhelm Olbrich. (Göschens Lehrbucherei.) Berlin, de Gruyter and Cod 1930. vi+254 pp.

In this one volume Professor Olbrich presents a revision of the two-volume work of Doehlemann, published in 1902. To condense the material into one volume, some topics in projective transformations have been omitted, as, for example, the linear transformations of a quadric surface into itself. The material of the second volume of the old edition, which was devoted to quadratic and higher birational transformations, has been cut down greatly. The new edition contains a brief exposition only of quadratic transformations in the plane and in space, inversion in the plane and in space, and simple transformations in the complex plane.

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