plete mathematical references are given in an appendix. The same is true of a number of other purely mathematical topics, such as complex integration, orthogonal functions, curvilinear coordinates, etc. A whole chapter is devoted to the study of hydrogen atoms by the wave mechanics method with sufficient concrete details and applications to make clear to the student the utility of the method.

The matrix mechanics is then introduced and developed as an independent theory, with detailed study of applications. This is followed by a discussion of the connection between the wave and matrix mechanics, and the method of constructing the quantum matrices from the solutions of the wave equation. In the chapter on the general theory of quantum dynamics, there is a thorough treatment of Heisenberg's indetermination principle, and the transformation theory of Jordan and Dirac. The frequent introduction of concrete illustrations greatly enhances the value of this chapter.

The book is concluded with chapters on the treatment of non-hydrogenic atoms and molecules by the new mechanics, spectral intensities and the diffraction of electrons and atoms by crystals.

The style of the book is in general clear and concise. The typography is excellent and the text is well illustrated by a large number of well-made diagrams. On the whole it is a work which may be heartily recommended to all those interested in the problems of atomic structure.

<div align="right">R. B. LINDSAY</div>

Leçons sur les Ensembles Analytiques et leurs Applications. By Nicolas Lusin. With a preface by Henri Lebesgue and a note by Waclaw Sierpinski. Paris, Gauthier-Villars, 1930. xvi+328 pages.

This volume in the Borel series contains a systematic survey of the present knowledge of analytic sets, a knowledge which is chiefly due to the researches of the Russian mathematician who is the author of this book. In fact the only results which are not due to Lusin or his pupils come from members of the Polish school of Sierpinski and Mazurkiewicz. The analytic sets of Lusin, which are a generalization of Borel sets, have been briefly mentioned previously in several books (Hausdorff's *Mengenlehre*, for instance), but this is the first book devoted entirely to their study.

Lebesgue in his preface humorously points out that the origin of the problems considered by Lusin lies in an error made by Lebesgue himself in his 1905 memoir on functions representable analytically. Lebesgue stated there that the projection of a Borel set is always a Borel set. Lusin and his colleague Souslin constructed an example showing that this statement was false, thus discovering a new domain of point sets, a domain which includes as a proper part the domain of Borel sets. Lebesgue expresses his joy that he was inspired to commit such a fruitful error.

Of the five chapters of approximately equal length into which the book is divided, the first two are devoted to Borel sets. Here and throughout the book, Lusin considers as his fundamental domain the set of irrational points of a linear space. By excluding the rational points, certain simplifications in statements and proofs of theorems are obtained. After mentioning several different methods of defining Borel sets and showing their logical equivalence, a study is

made of the structure of Borel sets from a geometric standpoint. Theorems are obtained describing the structure of a point set of a given class of Baire-de la Vallée Poussin. In particular, sets of class 0 and class 1 are described and the constructive existence of sets of classes 1, 2, 3, and 4 is shown. By showing the existence of universal elements in space of two dimensions, that is two-dimensional sets of class α such that any linear set of class α is obtained by cutting the set by a properly chosen line, the effective existence of linear sets of each class is shown.

After this introduction, the author shows in Chapter 3 how the concept of a Borel set may be generalized to that of an analytic set, and the properties of analytic sets are then studied in some detail. In this connection Lusin makes much use of the ingenious device which he calls a sieve (*crible*). The original notion of this device Lusin modestly attributes to Lebesgue, while the latter generously states that Lusin has read into Lebesgue's paper ideas which were not there but which are due entirely to Lusin himself. Be that as it may, Lusin shows by means of a sieve that Lebesgue has given in his 1905 memoir the first example of an analytic set not a Borel set that exists in mathematical literature. The reason for the friendly disagreement mentioned above is that there is a difference of view-point involved. Lebesgue was interested in analysis only, and hence did not attempt a geometric interpretation of the function which he had obtained in his memoir. Lusin, on the other hand, was interested in Lebesgue's example from a geometric standpoint and his geometric interpretation of it brings to light certain implications of which Lebesgue was not aware. Except for this example due to Lebesgue, the results of Chapter 3 are almost exclusively the work of Lusin and Souslin.

Then follows a brief fourth chapter on implicit functions, constituting the "applications" mentioned in the title of the book. Lusin shows how the problem of the domain of existence of a function of Baire is tied up with the geometric properties of that domain from the standpoint of analytic sets.

The final chapter is composed of two distinct parts, a study of projective sets and an analysis of the memoir of Lebesgue which has been referred to previously. In the analysis of Lebesgue's memoir, its connections with the present book are pointed out and Lusin's point-set point of view serves to complement Lebesgue's functional point of view.

The class of all projective sets forms a still more general class than that of the class of all analytic sets, and includes as well the class of all sets complementary to analytic sets. In this part of the book the results are due to Mazurkiewicz and Sierpinski rather than to Lusin, although the latter has contributed to this topic also. The book concludes with a two-page note by Sierpinski on a topic connected with projective sets.

The book as a whole is an admirable presentation of a subject in which research is now being actively pursued, and should prove invaluable to newcomers in the field. The author gives copious references to the literature for those who wish to pursue further the topics discussed. He also takes pains to point out the outstanding unsolved problems in the field, their implications, and their connections with present scientific knowledge in this domain of mathematics.

<div align="right">H. M. Gehman</div>