DIRAC ON QUANTUM MECHANICS

The Principles of Quantum Mechanics. By P. A. M. Dirac. Oxford, The Clarendon Press, 1930. x+257 pp.

This work may be divided into two parts, the first dealing with the general physical ideas of the quantum theory, the second treating the applications. The second part amounts to a collection of the author's well known papers on quantum dynamics which have appeared within the last half-dozen years in the Proceedings of the Royal Society. The presentation has been unified, and in many respects improved, though not without exception. It deals with the following topics: Elementary Applications; Motion in a Central Field of Force; Perturbation Theory; Collision Problems; Systems Containing Several Similar Particles; Theory of Radiation; Relativity Theory of the Electron.

It is the first part of the work which forms the essentially new contribution: it provides us with the author's conception of the fundamental ideas of the new physics, ideas which have only been implied in the works which he has published hitherto.

The principal concepts are the *states* of a system, the relation of *superposition* which may exist between states, and the *observables* which furnish information concerning the system. These notions are introduced on an a priori experimental basis, and only subsequently is a symbolism introduced for their expression, and an algebra for the formulation of their properties. Lastly, the conventional statistical interpretation is given. It is observed that this is the reverse of the customary order, in which the theory of operators comes first and the physical interpretation last, or more confusing still, is sprinkled in with the mathematical theorems. Yet we feel that our author has chosen the only proper order, either for gaining insight into the physical situation, or for giving the mathematical material its appropriate position. We only wish that he had been more thorough-going in this direction.

The style is crisp and vivid, and the intuition which the author displays and, in a sense, lends to the reader, is little short of profound. But in what concerns logical clarity of exposition, the work leaves much to be desired. For example, we are told in §§2 and 3 that a system (which may consist of one indivisible particle) the past history—or "preparation"—of which has been definitely observed to be A, "can equally well be considered" to have had part of it experience the past history B, and the other part the history C, where A, B, and C are all distinct. At this point we are reassured by recalling the author's earlier statement referring to the special case of the photon: "The description which quantum mechanics allows us to give is merely a manner of speaking which is of value in helping us to deduce and to remember the results of experiments and which never leads to wrong conclusions. One should not try to give too much meaning to it." By the end of the chapter one is enabled to cast the definition of superposition into final form, and to discover that it involves in an essential manner certain statistical concepts. In the second chapter, where the algebra of states and observables is introduced, such symbols as $=$, $+$, $-$, 0, etc., are ostensibly given a physical definition, and one emphatically not allowing them to be handled in the ordinary manner of algebra; nevertheless, they are so handled, and one arrives at the conclusion that what was ostensibly a definition was given merely for the sake of orientation, the actual definitions being supplied only by implication. It is our opinion that the difficulties of exposition in these and many cases of the sort are not fundamental, but could have been removed with a little care and the avoidance of the double use of terms.

On the mathematical side, the most notable feature of the work is the author's use of a space of a non-countable infinity of dimensions in which to represent his physical states. This space contains Hilbert space as a subspace, together with such further elements as the δ function and its derivatives. Although the axiomatics of this space are not given, and properties belonging to Hilbert space are carried over without justification, we believe the mere introduction of this space on a physical basis to be of great interest. As to the question of whether, if the logical work required for setting this space on a sound basis were actually to be carried through, the resulting treatment would present any advantages over the corresponding treatment by means of Hilbert space, we are unable to venture an opinion.

The notation and formal ideas are those of the transformation theory, according to which linear operators are regarded as being coextensive with infinite matrices. But von Neumann* has shown that the matrix representations of any two discontinuous operators (like those of quantum mechanics) are equivalent, in a certain sense, under a unitary transformation, a fact which should give us pause in accepting the transformation theory.

We feel that the usefulness of the book would have been enhanced by supplying it with an appendix, and by giving more references. It would have been well, in particular, to have added the name of von Neumann to that of Weyl among the exceptions to the statement that the expositions of quantum mechanics in the literature fail to take an abstract point of view and are always given by means of a definite system of coordinates (see the Introduction).† Finally, we object to the coining of the words "eigenvalue" and "eigenfunction," particularly since usage provides the terms "characteristic number" and "characteristic function," as the precise rendering of the German "Eigenwert" and "Eigenfunktion."

But if the *Principles of Quantum Mechanics* has its minor mathematical defects, I hope that my position as mathematical reviewer has not allowed me to obscure the fact that we have before us a work of no ordinary character, a work which by its depth of intuition, its originality of conception, and its power of insight, constitutes a notable advance in this department of thought.

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* J. von Neumann, Journal für Mathematik, vol. 161 (1929), p. 208.

† J. von Neumann, *Wahrscheinlichkeitstheoretischer Aufbau der Quantenmechanik*, Göttinger Nachrichten, 1927, p. 245.