

## CAJORI ON MATHEMATICAL NOTATIONS

*A History of Mathematical Notations.* By Florian Cajori. Two volumes. The Open Court Publishing Company, September, 1928, and March, 1929. xvi+451 pp.; xvii+367 pp. Per volume, \$6.

This work is a much more ambitious undertaking than merely to trace the historical development of mathematical notations now in use. The author states that his endeavor has been "to do justice to obsolete and obsolescent notations, as well as to those which have survived and enjoy the favor of mathematicians of the present moment." This enlarged program accounts for the appearance of two substantial volumes where a single and smaller volume might, perhaps, have been expected.

Volume 1 deals with notations in elementary mathematics, in three divisions: Numeral symbols and combinations of symbols, pages 2-70; Symbols in arithmetic and algebra, pages 71-400; Symbols in geometry, pages 401-431. In the first division, 45 pages are given to descriptions of the mathematical notations of the Babylonians, Egyptians, Phoenicians and Syrians, Hebrews, Greeks, Early Arabs, Romans, Peruvians, Aztecs, Maya, Chinese and Japanese, and the remaining 25 pages to the history of the Hindu-Arabic numerals. In the second division, the first 158 pages present the notations of more than 50 individual writers, representing many different nationalities, and the remaining 172 pages give a topical survey of the use of notations in various operations and relations of elementary arithmetic and algebra.

Volume 2 treats of notations, mainly in higher mathematics, considering them in four divisions: Topical survey of symbols in arithmetic and algebra (advanced part), pages 1-141; Symbols in modern analysis, pages 142-314; Symbols in geometry (advanced part), pages 315-326; Teachings of history (as related to mathematical symbolism), pages 327-350.

The two volumes constitute a notable contribution to the growing collection of American books on the history of mathematics. They seem to the reviewer to be the most valuable and the most scholarly work Professor Cajori has yet written. Both volumes should be in every college library and the first volume in every high-school library.

In spite of its great excellence, the work seems to be lacking in some respects. Even a casual reader would be likely to notice the scant attention paid to geometry. This neglect may be due chiefly to the nature of the subject so far as elementary geometry is concerned but the same can scarcely be said of the various fields in higher geometry. Only 30 of the 431 pages in volume one and only 12 of the 350 pages in volume two are devoted to geometry. Of the 126 illustrations in the two volumes only one small cut appears in a part given over to geometric notation, although several illustrations showing geometric notation are given elsewhere (in connection with individual notations, vol. 1, pp. 203, 207, 209, 210). In view of the fact that the second volume contains 71 fewer pages than the first (which does not appear to be bulky) the reader who is particularly interested in geometry is not likely to feel that the

author is entirely justified in saying (vol. 2, p. 315) "Lack of space prevents the enumeration of the great masses of symbols occurring in the extensive literature on the recent geometry of the triangle and circle." He is likely to feel something of mental protest at seeing  $13\frac{1}{2}$  pages of this second volume devoted to the "evolution of the dollar mark" (which may seem doubtfully classed as a mathematical symbol at all) and the symbolism of the illustrious Poncelet, generally considered the chief founder of projective geometry, dismissed (vol. 2, p. 319) with the two brief and somewhat misleading sentences: "Poncelet, in his *Traité des Propriétés Projectives des Figures*, uses little symbolism. He indicates points by capital letters  $A, B, C, \dots$ , and the center of projection by  $S$ ." The author cites only the first volume (Paris, 1865) of Poncelet's *Traité*. In the second volume (Paris, 1866) Poncelet employs considerable symbolism, for example,

$$\frac{(ap)(bq)(cr) \times aP \cdot aP' \times bQ \cdot bQ' \times cR \cdot cR'}{(bp)(cq)(ar) \times bP \cdot bP' \times cQ \cdot cQ' \times aR \cdot aR'} = 1,$$

which occurs on page 169; and he makes free use of small letters as well as capitals to represent points. Not only is there no attempt to trace the extensive symbolism of differential geometry and projective differential geometry, the subjects themselves receive no mention.

All references in the indexes and lists of illustrations are to paragraphs instead of pages. This manner of reference is convenient for an author, but wastes the time of the reader and is especially unfortunate in a book likely to be used largely for reference. If paragraph numbers had been printed in bolder-faced type or guide numbers placed at the tops of pages, as guide words are placed in dictionaries, this inconvenience would have been lessened materially.

A few typographical errors were noticed. In line 1, page 73, volume 1, "X<sup>3</sup>" occurs for "x<sup>3</sup>". In footnote 2, page 248, volume 1, N. J. Lennes is referred to as U. J. Lennes. A part of line 19, page 11, volume 2, reads "Then all the sudden he writes . . ." This is probably a misprint for "Then all of a sudden he writes . . ." On page 21, volume 2, a part of lines 25 and 26 reads "As seen in Figure 111, the same manuscript sometimes shows of widely different shapes." Such mistakes are, however, relatively few and such as occur are not very serious. References are abundant and carefully verified.

Obviously a history of mathematical symbolism which should cover all of the highly specialized fields of modern mathematics would have to be a work of collaboration. There are limits to human patience and endurance, also, and few readers are likely to have any adequate realization of the time and painstaking effort the production of these volumes must have cost their author. No minor considerations should allow a reader to forget that mathematicians owe a debt of gratitude to Professor Cajori for having written so excellent a work.

Recognition should be given to the Open Court Publishing Company for the high grade product of the printer's art shown in these volumes. Selection of type, quality of paper, press-work and binding are all eminently satisfactory.

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