SHORTER NOTICES

Applications of the Theory of Elliptic Functions to the Theory of Numbers. By P. S. Nazimoff. Translated from the Russian by Arnold E. Ross. Mimeographed by O. E. Brown. Evanston, 1928. 146 pp.

The original of this remarkable book was presented as a magister thesis in Russia and was printed in Russian at Moscow in 1884. Abstracts of certain parts of it were published shortly after its Russian appearance, both in the Fortschritte and in a prominent French journal. Neither abstract did the book justice. That this assertion of fact is so can be seen from the following. Had an adequate account of the contents of this book appeared at the time of its publication, much later work in the theory of numbers of representations of integers in special quadratic forms need never have been published. For example, in 1914, Bulyguin, using the methods of the first few chapters of Nazimoff's book, gave a complete solution, by comparatively elementary means, of the determination of the numbers of representations of integers as sums of any even number of squares. This work itself, like the neglected Nazimoff's, seems to have escaped the notice of the professional abstractors. As late as 1919, G. Humbert, in a letter to the present reviewer, inquired what had become of this most remarkable work of Bulyguin; it was already in print. Episodes such as this but emphasize the lamentable ignorance under which most of us labor concerning Russian work in the theory of numbers. Reverting for a moment to the question of representations as a sum of an even number of squares, we may state that many recent and elaborate investigations were out of date before they were printed, on account of the work of the neglected Russian school.

In the present translation only those parts of Nazimoff's book have been presented that contain matter which was original at the time of publication and a great part of which, in the light of the above remarks, is still as fresh as it was when published. The title sufficiently describes the contents of the work. Nazimoff made the first systematic attempt to obtain everything arithmetical that is implicit in the theory of elliptic functions. That such a vast undertaking appears only rough hewn in the final product, is not to the author's discredit. Precisely where his work is least finished is the place where it is most suggestive. The problem of discovering everything that is implied for the theory of numbers by the multiply periodic functions is a well-put and solvable one; this book took the first significant step toward a solution. It is worthy of the attention of all those interested in the algebraic applications of analysis to the theory of numbers—which is quite a different thing from the analytic theory of numbers.

For those interested, the general problem which will be suggested by a reading of pages 138–144 of the present translation, is worthy of the closest attention. A complete solution of the obviously implied problem, extended to functions of any number of variables, contains the solution of any arithmetical problem whatever that is solvable by the application of algebraic functions of elliptic theta functions.

The translation was made from what is apparently the unique copy of this book in the United States. The translators wish it to be stated that copies can be purchased only at the University of Chicago Book Store, 5802 Ellis Ave., Chicago, and that the price is $3.00. The 146 pages of mimeographing are, on the whole, clearly done; the book is substantially bound in a stiff paper cover and, as such work goes, is of a very high quality. It is to be hoped that this translation will at last make Nazimoff's penetrating ideas familiar to those who either do not read Russian or who are content to accept second hand abstracts from those who also do not read Russian.

E. T. BELL

The new Quantum Mechanics. By G. Birtwistle. Cambridge University Press, 1928. xiii+290 pp.

This book contains an exposition of the theories of Heisenberg, Schrödinger and Dirac, with applications mainly to the theory of spectra. There are also chapters on de Broglie's particle waves, and the Bose-Einstein and Fermi-Dirac statistics. The closing chapter deals with Heisenberg's indetermination relations and the formulation given to them by Bohr in his article in Nature, April 1928 (and communicated to the author before publication; the author's preface is dated Copenhagen, 1 Oct. 1927).

The book contains a great deal of information on the main developments in quantum theory up to the latter part of 1927, and is convenient for reference; on the other hand, owing perhaps to lack of time, the presentation is uncritical, so that the book cannot replace the study of the original memoirs.

T. H. GRONWALL

Traité d'Analyse, volume 3, third edition. By Émile Picard. Paris, Gauthier-Villars, 1928. ix+660 pp.

This is a reprint of the second edition, with additional notes on pp. 601–656. Although no review of previous editions has appeared in the Bulletin, the *Traité* is so well known and so highly appreciated by the mathematical public, that it seems superfluous to describe the contents of the main body of the volume. The additions to the new edition are as follows: (1) a note on Sundman's work on the problem of three bodies (reprinted from Bulletin des Sciences Mathématiques, 1913, pp. 313–320); (2) a lecture on integral invariants and Poisson stability (from the author's course at the Sorbonne in 1914, printed here for the first time); (3) remarks on some of Poincaré's results in analytical mechanics (Bulletin des Sciences Mathématiques, 1914, pp. 320–236); (4) on the solution of $\Delta u = e^u$ on a closed Riemann surface (Journal für Mathematik, vol. 150); (5) on linear partial differential equations and the generalization of Dirichlet's problem (proof of the analyticity of the solution for a second-order equation of the elliptic type, reprinted from Acta Mathematica, vol. 25 (1901), pp. 121–137).

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