SHORTER NOTICES

The Bakhshâlî Manuscript—A Study in Mediaeval Mathematics. By G. R. Kaye. With photographic facsimiles of the manuscript and transliteration of the text in roman script. Calcutta, Government of India, Central Publication Branch, 1927. Pp. i+156+v. Plates 1+4+42. Price Rs 28 or 43s. 6d.

Bakhshâlî is the name of a village near the city of Peshawar in the north-west corner of India. There in 1881, a farmer unearthed a manuscript of a work on mathematics. The greater portion of it has been destroyed by rough handling and the remains consist of 70 leaves of birch-bark but some of these are mere scraps. That manuscript is now generally known as the Bakhshâlî Manuscript. It is preserved in the Bodleian Library, Oxford. The work of analysing and editing the Manuscript was begun by the distinguished Indologist, Dr. R. Hoernle, but on account of his untimely death, it came to the hands of Mr. G. R. Kaye who has successfully finished it. The editor has added an elaborate and masterly introduction to the text covering 84 pages in print which contains not only an analysis of the contents of the manuscript but also a study of its relation with other Hindu treatises of mathematics as far as possible.

The Bakhshâlî work is a compendium of rules and illustrative examples. It gives also the solutions of the most of the examples together with their verification. It contains material relating to arithmetic, algebra, and geometry (including mensuration). The topics for discussion are found to include fractions, square-root, arithmetical and geometrical progressions, income and expenditure, profit and loss, computation of gold, interest, rule of three, summation of certain complex series, simple equations, simultaneous linear equations, quadratic equations, indeterminate equations of the second degree of a particular type, mensuration and miscellaneous problems. The treatment of all these subjects is commonly included in other Hindu treatises of mathematics. But we miss in it any reference to the treatment of the indeterminate equation of the first degree and the so-called Pellian equation, both of which enter largely into later Hindu works and in the solution of which the Hindus long anticipated the works of Euler and Lagrange.

The most notable mathematical principles in the Bakhshâlî work are the approximate square-root formula,

$$(a^2 + r)^{1/2} = a + \frac{r}{2a} - \frac{(r/(2a)^2)}{2(a+r/(2a))},$$

the calculation of the errors of successive order

$$\epsilon_1 = (r/(2a))^2, \quad \epsilon_2 = \left(\frac{(r/(2a)^2)}{2(a+r/(2a))}\right)^2$$

and a consequent process of reconciliation. The fundamental arithmetical operations are indicated by putting after, occasionally before, the quantity affected, the abbreviations of the Sanskrit words of respective import. In the case of subtraction, the abbreviation is found to have degenerated into a simple cross-sign (+). The Bakhshâlî work is particularly characterised by the absence of any kind of algebraic symbolism. This has necessitated the preservation of every detail of the working of the solutions of algebraic problems keeping up their generality throughout so that the final statement of the results should clearly present the whole formulas involved. "Indeed the numerical quantities in those problems are treated almost like algebraic symbols," rightly observes Mr. Kaye. Most of those problems have been solved by the method of *false position*. The place of an unknown quantity in a statement of a problem is indicated by a heavy dot (•) and is designated as *śûnyasthāna* or "empty space," meaning thereby that the number to be placed there has not been ascertained. Other matters of interest are: the least common multiple, the solution of the quadratic equation, the exclusive use of the decimal place-value notation with the cipher, the average value and the change ratios of measures.

The time of the Bakhshâlî work is rather uncertain. On paleographical evidence Mr. Kaye holds that it should be referred to a period about the twelfth century. There are clear signs to show that the present manuscript is an imperfect copy of an older manuscript which again is of the character of a perpetual commentary on a still earlier work. The editor has failed to detect it. But it did not escape the keen eyes of Dr. Hoernle. He is of opinion that the present copy was made about the ninth century but the original work must be referred to the early centuries of the Christian era. There are many things in the work to point to the same period. Mr. Kaye has drawn attention to a few instances of foreign influence in the Bakhshâlî work including the one about the approximate square-root stated above which he says he has not found in any other Hindu works. That formula, correct up to the second approximation, was known in the early Jaina canonical works (about 300 B.C. or still earlier) and also reappears in later works including that of Âryabhata (499 A.D.) as has been pointed out by Rodet. Another has been overstated under a similar misapprehension and the rest are minor and of doubtful value. The editor himself does not wish to press upon the latter.

The significance and importance of a publication like the present one are apparent to all lovers of the history of science. And their thanks are certainly due to Mr. Kaye for the great amount of pains that he has taken in explaining and editing the Bakhshâlî manuscript. There are, no doubt, several inaccuracies in his study of the work, some of which are not negligible. It is not possible to point them out in a short review like this. A critical examination of them will be found in a paper by the present reviewer entitled *The Bakhshâlî mathematics* in the Bulletin of the Calcutta Mathematical Society (vol. 21, pp. 1–60). But they do not lessen the labor of the editor. Moreover such inaccuracies are not unusual in case of all pioneer studies of difficult ancient manuscripts.

BIBHUTIBHUSAN DATTA