

The reviewer believes that the book will be a welcome addition to the treatises on the subject. Mr. Miles and those responsible for the Pamphlet are to be congratulated upon the careful way in which the lectures are edited. Misprints are rare, and the general high character of the Pamphlet, established in earlier numbers dealing with mathematical subjects, is maintained. The consistency with which references are given to unproved theorems and to the proofs of facts employed in the demonstrations is highly commendable. If this principle were followed more generally, mathematical literature would be more readable.

D. V. WIDDER

*Theory of Probability.* By William Burnside. Cambridge University Press, 1928. 106+xxx pp. Short table of Error-Integral.

This *Theory of Probability* was edited for the press by A. R. Forsyth from a manuscript which Burnside practically completed before his death. In barely a hundred pages, it treats a great variety of topics in a decidedly unique and interesting manner. It is not a book on statistics. Burnside's interest is in probability itself. First, he develops in somewhat extended form relations between probabilities of a general nature associated with  $n$  conditions, some of which are to be satisfied and some not—connecting his primal idea of equal likelihood with these conditions. Then he proceeds to the discussion of typical problems, mainly algebraic in their origin. Analysis is freely used—of course, it is indispensable—but the reader gets the impression that Burnside tries to view a problem first in an algebraic setting. Thus the difference equation is more in evidence than the differential equation. This is a distinctive feature. It makes the book of special value to readers who have cultivated mainly the analytic side.

The book is fairly well organized, in spite of the fact that Burnside may not have considered it in its final form. A greater unification of topics involving the Gaussian law would have been desirable,—this law appears on pages 42, 44, 52, 73, 88. On page 73, the validity of this law as the best approximation is not so clear, as the elementary probability designated by  $x$  is made very small—this leading naturally to the Poisson Exponential, obtained on page 45. Burnside is not satisfied with the usual statement of the assumption of equal likelihood. He insists upon “assuming each two of the  $n$  results equally likely,” instead of “assuming all the  $n$  results to be equally likely,” see page 101. Burnside's phrasing appears a little more specific—although it seems difficult to conceive how all can be equally likely if some two of the results are not equally likely. In Chapter VII, which deals with the theory of errors, assumptions are given which lead respectively to the arithmetic mean, the median, and the average of the least and the greatest measurement. The arithmetic mean is found acceptable “as any other assumption would imply that either an excess of positive errors or an excess of negative errors has occurred.” If here by “positive errors” the *number* of positive errors is meant, the median would be indicated rather than the arithmetic mean. The argument in Chapter VI follows conventional lines, including an artificial assumption regarding the a priori

probability at the top of page 85, made so as not to violate the first equation on page 84. Not many disturbing typographical errors were found—but the radicals at the foot of page 89 and at the top of page 90 should be in the denominators, and on page 91, the coefficient of the square of  $X$  should be inverted. On page 64 in the last line,  $s/2$  should be replaced by  $(s/2) - 1$ , and  $(s/2) - 1$  by  $(s/2) - 2$ . Here and at the foot of page 98 mention might have been made of Pearson's *Tables of the Incomplete Gamma Function* for computational completion of the problems. At the foot of page 35 it is assumed that  $p$  does not equal  $1/2$ .

To epitomize the content of Burnside's book is somewhat difficult. The last chapters deal with the probability of causes (with interesting exemplification of Bayes's Theorem), geometrical probability, and the theory of errors. As early as page 22, problems on "runs" appear, and these under the title "duration of play" constitute the principal part of Chapter III, and are again taken up on page 72. These problems are more fundamental than they at first seem. They are especially fascinating in a book so full of good material.

E. L. DODD

*Drei Abhandlungen über die Auflösung der Gleichungen* von Leonhard Euler (1738, 1764, 1790). Translated by Samson Breuer. 94 pp. Leipzig, Akademische Verlagsgesellschaft, 1928. Ostwald's Klassiker, Nr. 226.

Students interested in the history of the theory of equations will find here, reproduced in translation from the Latin into German, three treatises by one of the greatest mathematicians of the eighteenth century, who still held to the belief that the algebraic solution of the general quintic equation and equations of still higher degrees was possible. Knowing the extraordinary dexterity of Euler in analytical processes, we are impressed by his statement that though his article did not contain a complete solution, it indicated the form which a root must take, and perhaps it would be "of greater service to others and lead them finally to the desired goal." Euler continues: "Since an equation of any degree comprehends all equations of lower degree, the solution of an equation of any degree must involve all the processes for equations of lower degree." Euler assumed as given by induction that the resolvent of the general quintic is a quartic. In the second treatise Euler remarks that if  $p, q, r, s$  are the roots of the quartic resolvent, then the general form of a root of a quintic equation would seem to be  $x = f + \sqrt[5]{p} + \sqrt[5]{q} + \sqrt[5]{r} + \sqrt[5]{s}$ , but he remarks that this form is not sufficiently sharp as it would seem to lead to more than five roots of the quintic. Euler assumes next  $x = f + A\sqrt[5]{p} + B\sqrt[5]{p^2} + C\sqrt[5]{p^3} + D\sqrt[5]{p^4}$ , and is led to solvable equations, including those previously solved by De Moivre. In the third article, written in 1776 and printed in 1790, Euler assumed the coefficients to depend upon two or more parameters and then derived numberless forms of equations of all degrees, which are solvable by algebra. Breuer, the translator of these articles, has supplied much bibliographical detail and many notes aimed to assist the reader.

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