SHORTER NOTICES

Collected Papers of Srinivasa Ramanujan, edited by G. H. Hardy, P. V. Seshu Aiyar and B. M. Wilson. Cambridge University Press, 1927. xxxvi+355 pp.

Ramanujan's great and extremely individual work is now too well known to all who specialize in the theory of numbers to require detailed comment by any reviewer. The editors of his works have reprinted thirty-seven of his papers, each one of which merits the closest attention by students of algebra, the theory of numbers, and analysis. In addition they have added two extremely helpful appendices, the first consisting of notes on the papers, the second of further extracts from Ramanujan's letters to G. H. Hardy. Ramanujan's brilliant and tragic career is feelingly portrayed in the introduction. Without doubt, he was one of the few great mathematicians who have had a superlative genius for numbers, and it is indeed a tragedy that he, like Eisenstein, had to leave the world so early.

To attempt a detailed review of any of these great papers would be impertinent. We may glance, however, at one or two human problems raised by the general color of Ramanujan's brilliant work. The first of these concerns editors of mathematical journals. To the reviewer it is incredible that certain of these papers would have been accepted for publication in at least one of our American periodicals, and the like holds for more than one European journal. It is easy to deny this after Ramanujan's genius has been accepted and established. Yet on these same doubtful papers the stamp of genius is flamingly apparent to all who are not blighted by the rot of academic rigor. What does it matter, some may say, that many of his most original ideas are not backed by anything that even faintly resembles proof, when these same ideas of Ramanujan have already initiated what our successors may perhaps look back on as the first golden age of the analytic theory of numbers? The critical reader will recognize the justice of these remarks on a careful study of those papers designated by Professor Hardy as Ramanujan's greatest. To these may be added number 20, *On the expression of a number in the form* $ax^2+by^2+cz^2+du^2$. If this be sifted to the bottom, it will be found that little, if anything, is *proved*. The defects, of course, have since been supplied by later writers. Still, if proof means anything in mathematics, it should surely mean something in the theory of numbers above all other branches.

A second observation is this. The one theorem, or formula, selected by Major MacMahon as the most beautiful in all of Ramanujan's work, is indeed a thing of beauty. But, as recognized by the editors, this, and many more of equal elegance, are already implicit in the neglected papers of Professor L. J. Rogers. This assertion is not meant to detract in any way from the brilliancy of Ramanujan's totally independent rediscovery of a beautiful theorem. It is meant to direct the attention of young mathematicians to much formal, unfashionable, and unadvertised work on the algebraic side of the theory of numbers which has been overlooked. Without the great publicity which Ramanujan's work has given to these neglected researches, it is doubtful whether they would have survived oblivion. The conclusion of the whole matter is this: the man who is capable of reading will neglect the official and perfunctory abstracts of work that are ground out yearly in the reviews, and glance through the papers as printed in the journals themselves.

Ramanujan has been likened to Jacobi. To the reviewer he seems also to be akin to Eisenstein, Hermite in his earlier work, and to Galois, for the boldness of his thought. Whatever may be the ultimate estimate, it seems reasonable to predict that Ramanujan will be placed high among creative mathematicians. Above all he was an algorist and an arithmetician of the first rank. The brilliance of his papers of the later period, written under the influence of the English school, which is today transforming the analytic theory of numbers, give us a just measure of our loss in the death of a man at the age of thirty-two who might also have been one of the world's great analysts.

E. T. Bell

Encyklopädie der mathematischen Wissenschaften. Volume III. Part 3. Leipzig, Teubner, 1902–1927.

Of the ten articles which compose this volume of more than 800 pages, mostly devoted to differential geometry, some were first published as early as 1902, some as late as 1927. As a result there is a certain inhomogeneity; it is probable that the earlier parts which cover the classical theory of curves and surfaces and which occupy about one half of the volume already have rendered the greater part of the services as a reference book of which they were capable and a more up to date reference book reflecting the advances made in the eventful last quarter of century seems desirable. Fortunately, the articles by Weitzenboeck and Berwald in which differential geometry of n dimensions is covered—a field that has received a great deal of attention lately, partly under the influence of the theory of relativity—bring the literature up to 1923. Differential geometries under groups other than the metric also are dealt with in these two articles. The references are very complete; if there are omissions they probably occur only in cases of dissertations which did not appear in a periodical (I noticed one such case). These two articles are of inestimable value to one who works in the field.

In addition to the articles mentioned, there is a comparatively recent article on triple orthogonal families (Salkowski) and two articles by Liebmann (dated October 1914) which are but loosely connected with differential geometry proper. One is devoted to contact transformations and the other to the geometrical theory of differential equations; what is meant here is the line of attack on differential equations by means of analysis situs considerations started by Poincaré.

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