KLEIN ON NINETEENTH CENTURY MATHEMATICS

Vorlesungen über die Entwicklung der Mathematik im 19. *Jahrhundert.* By Felix Klein. Teil I. Berlin, Springer, 1926. xiii+385 pp.

Twenty years ago, during a series of walks in the forest about Hahnenklee, in the Hartz Mountains, the conversation between Klein and a companion covered, as would naturally be the case, a wide range. Three statements, however, impressed his listener* very strongly. One was political: "There was a time when we looked up to England socially, politically, and as a naval power,—but that is a thing of the past." The second was political and of military significance: "America has no standing army today; twenty-five years hence she will have a large one." It is not strange that his auditor wondered at the real significance of these two statements by a man of Klein's vision and prominence. The third remark was in response to a statement to the effect that he of all men was the one to write a history of mathematics in the 19th century. "I am too old," was the reply, "It needs a young man who could devote years to its preparation." When it was urged that he had seen the development and had taken part in it as few if any others living had done, he remarked, "No, all that I could do would be to give a few lectures on the great events, but I am too much occupied to prepare even these." Ten years later, when the war was on, and his family had been sorely stricken, he gave these very lectures in his home in Göttingen, before a small group of listeners anxious to receive from a master that which only a master could give.

The lectures have been edited by Professors Courant and Neugebauer and are published as Band XXIV of Die Grundlehren der mathematischen Wissenschaften, a recent series, already well known to all mathematical students. Rightly did they say in their Vorwort: "Diese Vorlesungen sind die reife Frucht eines reichen Lebens inmitten der wissenschaftlichen Ereignisse, der Ausdruck überlegener Weisheit und tiefen historischen Sinnes, einer hohen menschlichen Kultur und einer meisterhaften Gestaltungskraft; sie werden sicherlich auf alle Mathematiker und Physiker und weit über diesen Kreis hinaus eine grosse Wirkung ausüben."

The work is divided into eight chapters. The first naturally begins with the founder of the modern German school of pure and applied mathematics, —Gauss. It considers his work in the applied field with respect to astronomy, geodesy, and physics, the last in connection with A. von Humboldt and Wilhelm Weber. In pure mathematics the attention is given chiefly to the number theory, forms, and the function theory, with a succinct statement as to the claim that Gauss is entitled to the award of priority

* The reader may suspect the identity of the "listener," who modestly conceals it. The editors may state at least that the conversation was actual and that this report is a first-hand report.

(The Editors)

in the discovery of non-euclidean geometry, although not to priority of publication (pp. 57–60).

Chapter II relates to France and the Ecole Polytechnique in the first decade of the century. In this Klein pays high tribute to the work of men like Fourier, Cauchy, Poncelet, Monge, and Galois. Of Cauchy he speaks as one "der sich mit seinen glänzenden Leistungen auf allen Gebieten der Mathematik fast neben Gauss stellen kann."

Chapter III concerns the founding of Crelle's Journal and the rise of pure mathematics in Germany. In this period the names mentioned as most prominent are those of Abel, Jacobi, Moebius, Plücker, and Steiner.

Chapter IV considers the development of algebraic geometry by Moebius, Plücker, and Steiner, with reference to Lagrange, Chasles, Cayley, Sylvester, Salmon, Beltrami, and Clifford, as well as to his own countrymen,—Riemann, Hesse, and Grassmann.

Chapter V deals with mechanics and mathematical physics in Germany and England before 1880,—the period of Hamilton, Thomson (Kelvin), and Maxwell in England, of Gibbs in America, and of Franz Neumann in Germany.

Chapter VI is devoted to the development of the theory of functions of a complex variable, chiefly at the hands of Riemann and Weierstrass, but with mention of the influence of Dirichlet, H. A. Schwarz, Fuchs, C. Neumann, Kovalevski, and others.

Chapter VII is taken up with the study of algebraic forms, and Chapter VIII with that of the theory of groups and automorphic functions, with special reference to the work of Galois, C. Jordan, Hermite, Riemann, and Poincaré.

It is proposed, in Band II, to treat chiefly of the theory of invariants and of relativity.

All who knew Professor Klein with any degree of intimacy are aware of his broadmindedness and catholicity of spirit. His love for science did not permit him to allow political prejudices to warp his judgment as to the tribute due to scholarship beyond the boundaries of his own country. It speaks well for his judicial spirit to observe that, of the scientists mentioned, nearly half were not of German nationality. Nearly a fourth were French, about an eighth were British, and a fair number were from Italy, Scandinavia, Russia, and our own country. It was natural that the work of the Germans should have been best known to him, and it was entirely fair that he should have ranked them higher than others of the last half of the century, not alone numerically but on the score of achievement.

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