MOULTON ON EXTERIOR BALLISTICS

New Methods in Exterior Ballistics. By F. R. Moulton. Chicago, University Press, 1926. vi+257 pp.

This book is the outgrowth of Professor Moulton's work for the Ordnance Department of the U. S. Army during the World War and, to some extent at least, the result of his teaching the subject since that time.

The importance of the book is not questioned and an effort will be made properly to evaluate it in this review, with the definite understanding that statements made are merely opinions. But I wish to point out two aspects of work on ballistics in this country, pertaining to this book and its contents, that are somewhat disappointing.

Toward the close of the World War, and even during the period of hostilities, every person who made any pretense of being interested in contributions to mathematics longed to secure a book on ballistics from which he could compute a trajectory; not by the old antiquated methods which, he had heard, had been entirely discarded, but by the methods that had been devised by American mathematicians who had patriotically devoted their talents to the solution of these important problems. Except for a very few who were in one way or another officially connected with the Army, those who desired such a book at that time or even now have been doomed to disappointment. After two books have been published on the subject, one by R. S. (then Captain) Hoar in 1922, and the book under review by Professor Moulton, the student and the professor of mathematics have at their disposal no book from which a trajectory can be actually computed. Both books are devoid of tables, and without tables, a trajectory can no more be computed by the methods they have devised than a spherical triangle could be solved without the proper tables. From the standpoint of interesting persons in the subject of ballistics this is to be regretted. Such tables are regarded as confidential information by the U. S. Army. Regarding the propriety of this attitude the reviewer has no comment to make and no opinion to express. But it is proper, I think, to warn the reader that this withholding of the tables from not only public examination (to which there may be objections) but from the perusal and use of mathematicians, physicists, and engineers might give to such tables an importance that the facts do not justify.

The second disappointing feature, in the opinion of the reviewer, is of a more serious character. There has been a failure on the part of some properly to appraise the work done in this country during the War by our mathematical men who were interested in ballistics. The contributions that they made to the subject were advertised in most complimentary terms at meetings of the national mathematical organizations and elsewhere. No one was disposed to question them; no one was in a position to question them. It came to be usual to say certain things about these researches in ballistics. The failure adequately and fairly to appreciate the exact worth of this work is embodied in the following quotation from another recent review of Professor Moulton's book: "The fundamental differential equations of motion of a projectile in flight have been known since the days of Euler. Up to the World War the progress in ballistics had consisted in devising approximate algebraic expressions whereby to wrench these equations into soluble form." "These approximations had been improved and improved, but still could not keep pace with the development of modern artillery. So finally Professor F. R. Moulton of the University of Chicago, while serving as a Major in the U. S. Army during the World War, cut the Gordian knot by going back to the Eulerian equations and solving them in their original and exact form by numerical integration. Thus he laid the cornerstone for an entirely new science of ballistics." Moreover, the same reviewer in his following paragraph compares Professor Moulton's scientific achievement to that of Morse and Bell, and states that he should be given equal credit.

There is little doubt that the above quotation is an expression of confirmed opinion on the part of its author. Also it must be said that it embodies the attitude that has become all too prevalent regarding this work and which has not been corrected. Since I have said that this has not been corrected, let me give what I consider a more accurate appraisal of the same work: it is understood that this is my opinion and nothing more. (1) Solutions of these equations by numerical integration are just as truly approximations as solutions obtained in different ways by such men as Cranz, Richmond, Vahlen, Dufrenois, and others, all of whose methods were either developed or recast from other previously devised methods during the same period. Research in ballistics was not confined to this country. Many problems were investigated and at least partially solved abroad before they were undertaken on this side of the water.

(2) Numerical integration like any other method of successive approximation, if it is to become accurate, must be corrected from time to time. For instance, the square root or any rational root of an integer may be obtained by successive approximations because the number itself serves as a check upon the error made. But unless there is some means by which the errors of approximation are corrected, there is no guarantee that accuracy will be obtained, no matter how carefully the work may be done. This is exactly the case in the case of integrating the differential equations of the motion of a projectile. The only check is a tabulated function G which is built upon observed data. Although these data are the most reliable data obtainable, they are subject to errors of various kinds. The particular table referred to, according to all the evidence that can be obtained (and it is not easy to obtain) is built upon Gavre firings, firings at Aberdeen, upon interpolation, and upon some extrapolation. The point to be made here is that a trajectory computed upon this table as a basis could be no more accurate than the table itself. And even more important than this, if one had more reliable tabulated data, he might be able to obtain the results desired without so much trouble.

(3) It is admitted by all that the differential equations of the motion of a projectile in flight are insoluble by purely mathematical methods. The

best that can be done is to approximate such solutions. Thus the problem, from the standpoint of the practical ballistician at least, is one of engineering mathematics. If these premises are correct, then what is to be gained by making the mathematical method of approximation more difficult than is neccessary? If the reviewer has acquired the point of view of other ballisticians, such as Cranz, Dufrenois, and Vahlen, it is briefly summarized in this: Let us aim to get the physical data or tabulated data as accurately as possible, and the method of approximation as simple as possible. Surely this is the practical point of view, and it must be admitted to be preeminently the case when one takes into account the kind of educational training usually possessed by army officers of any nation. Their training must be broad and practical in the extreme, and they want mathematics presented in as simple and usable form as possible.

(4) The argument often presented to show the great advantage of methods of numerical integration in solving problems in ballistics over any other method of approximation is something like this: Other methods of approximation are practical up to a certain point but by methods of numerical integration results can be made as accurate as we please (by making intervals sufficiently small). This is far from correct. This statement involves several assumptions of a fundamental character. The method of numerical integration as outlined and perfected by Professor Moulton and his associates can be used to compute trajectories *as accurately as we please* only upon the following conditions:

(a) That the G table is absolutely correct.

(b) That the H table is absolutely correct.

(c) That the characteristic motion of a projectile is completely defined by its ballistic coefficient.

None of these is absolutely correct. The H table is highly satisfactory; the G table, although probably the best available, is subject to large errors; the ballistic coefficient only approximately characterizes the motion of a projectile.

In Chapter I the problem is outlined and the translatory motion of the projectile is described; the rotation of the projectile is purposely omitted from this discussion for the sake of simplicity and because rotation is the subject that lies at the basis of the last, and, in my opinion, the most important chapter of the book. It is particularly important to point out the kinds of things that might effect the motion of a projectile and especially the degree of accuracy that is needed. This is admirably done in Chapter II. The method of numerical integration is given in Chapter III, presented as clearly and as attractively as it could be done without the use of tables. The subject of differential variation as presented in Chapter IV will be interesting to the mathematician, but is not elementary, and in my opinion, is not given in a sufficiently elementary and practical manner. The adjoint system of differential equations is introduced as if it were such a familiar portion of mathematics that this could be done without explanation. Chapter V is theoretical in the extreme. It is devoted to a theoretical substantiation of the validity of the process of numerical integration, based upon the work of Picard. This is most attractively presented from the standpoint of logical sequence, but, as Professor Moulton suggests, this chapter should be omitted by those who are reading the book for practical purposes only.

In Chapter V, which occupies eighty-five of the two hundred fifty-seven pages of the book, Professor Moulton has set forth the mathematical basis upon which modern advanced ballistics reposes. This study of the rotating projectile is of fundamental importance in ballistics and especially in projectile design. Projectiles are constructed in such a way that both heads and tails may be varied by screwing such portions of different designs upon the body of the projectile. Moreover by screwing a cylinder to different positions within the body of the projectile, its moments of inertia may be varied. These projectiles are then fired through cardboard or beaverboard and their periods of precession, yaw, etc. may be studied. Although this has been added to since the War, it is the best presentation of the subject in our language, or in any language, that I have seen. The study of projectile design, and in general the problems of the rotating projectile, may be studied through the mathematics of this chapter as a tool, provided that experimentation can be conducted as it should be conducted. In this sense Professor Moulton has put the subject of ballistics upon such a basis that no essential change will have to be made in the mathematical method of attacking such problems for some time to come.

There have been three outstanding products of the work of our ballisticians and mathematicians during the War. The translation of the A. L. V. F. Tables; the book published by R. S. Hoar in 1922; and the book under review by Professor Moulton. In my opinion the most practical product is the translation of the A. L. V. F. Tables; they are just what the ballistician wants and needs. I doubt seriously that we could construct better or more useful tables. Placing ballistic research upon a sound mathematical basis is also a matter of great importance. Credit for this must be given to Professor Moulton and his associates. Also, they must have credit for giving an impetus to mathematical research in the Army and Navy that has had excellent results. This book records the mathematical methods of attacking problems in modern ballistics as developed in this country during the period of the World War. The same problems have been attacked and solved in slightly different ways by ballisticians in other countries during the same period. Greater accuracy in such work from the practical standpoint will depend upon more accurate physical data rather than upon new methods of mathematical attack. Although Professor Moulton's book is by far the best book on modern ballistics published in this country, it is for the use of the mathematician rather than for the practical ballistician. In the reviewer's opinion, it is not Professor Moulton's outstanding contribution to mathematical science. We still need a practical book on ballistics, provided with adequate tables, for the use of the officer, the teacher, and the ballistician.

J. E. Rowe