The Byzantine Astrolabe at Brescia. By O. M. Dalton. New York, Oxford University Press (American Branch). 14 pp.+3 plates. Price 70 cents.

This monograph was communicated to the British Academy on July 7, 1926, and is now reprinted from the Proceedings. The astrolabe described in the paper has an unusual interest because it is of Byzantine workmanship and dates from a period of which, as regards the science of astronomy on the banks of the Bosphorus, we know but little. The instrument is larger than most of those which have come down to us, being nearly fifteen inches in diameter, but the workmanship is crude in comparison with the neat work of European craftsmen of the Middle Ages or with the Arabic pieces. A special interest, however, lies in the fact that the inscriptions are all in Greek, the instrument being, as the author states, "perhaps the sole survival of an astrolabe representing a Greek type apparently unmodified by foreign ideas." On the arachne are some iambic verses which state that the piece,

> "being an intricate work, with ardent mind fashioned Sergius the Persian, holding a consul's rank";

and another inscription reads

> "Decree and command of Sergius, protospatharios, consul, and man of science [?], in the month of July, fifteenth indiction, year 6570,"

being the year 1062 of our era.

The author prefaces his monograph with a brief statement as to the general nature of an astrolabe and then gives description of the one which is the subject of his researches. The instrument is now in the Museo dell'Età Cristiana at Brescia, having been presented in 1844. The essay is a genuine contribution to our knowledge not only of the history of the astrolabe but of the state of science in the eleventh century.

DAVID EUGENE SMITH

Les Groupes Abéliens Finis et les Modules de Points Entiers. By Albert Chatelet. Paris and Lille, 1925. 221 pp.

As the author states in the introductory chapter, this book was the outgrowth of a much more limited treatment of abelian groups that he had originally planned as the first part of a treatise on abelian algebraic fields. While much more narrow in scope than the existing texts on groups, which treat those of the commutative sort more or less incidentally, it neverthe less embraces considerable material that is not found in such texts and, while following in large part the developments of the subject as given in papers by various authors, appears nevertheless to have considerable novelty in its treatment.

When the elements of an abelian group are represented in all possible ways as products of powers of a suitably chosen set, the set of exponents of such powers may be regarded as the coordinates of points in a certain number of dimensions. The set of points that thus correspond to the identical element of the abelian group form a "module" of a particular type in the same space, i.e., the "sum" or "difference" of any two points in the set is itself in the set. More generally, any element of the abelian group corresponds to a set of points with integral coordinates that are "congruent" with respect to this module. Following the method employed by Frobenius and Stickelberger in their well known paper on abelian groups, the author therefore develops their properties very largely from the corresponding properties of modules.

The problem of making a change in the "basis" of a module, i.e., a finite number of points in terms of which all the points of the module may be represented by means of integral multipliers, involves naturally the question of the equivalence of matrices of a somewhat more restricted sort than that employed in the usual algebraic theory. The proof of the existence of a "reduced" basis is omitted (rather unfortunately, it seems to the reviewer), reference being made to another text by the same author. The questions of "divisibility," "greatest common divisor," and "least common multiple" of modules and matrices are treated quite fully. The same is true of the subject of the "invariants" of matrices with integral elements, which bears rather a close analogy to the more familiar theory of elementary divisors.

Following the rather extended treatment of modules and matrices in the second chapter, the remainder of the book is devoted to abelian groups. The subsequent chapters vary considerably in interest and importance, the subject of the automorphisms of an abelian group being treated quite comprehensively, while the theory of group characters, being included in the corresponding theory for finite groups in general, seems too special to attract the reader particularly. This would have been unavoidable, however, unless the scope of the book had been considerably widened.

The book should prove valuable as a reference text on the subject of which it treats, although too specialized probably to have a very wide use. It has evidently been very carefully prepared, including as it does a complete index of definitions and a fairly comprehensive bibliography. In the text definitions are emphasized by bold-faced type and theorems are in italics. Some use of numbered formulas might have been an improvement.

H. H. MITCHELL

Höhere Mathematik. Teil I. *Differentialrechnung und Grundformeln der Integralrechnung, nebst Anwendungen.* By Rudolf Rothe. 2d edition. Leipzig, B. G. Teubner, 1927. vii+186 pp.

The first edition of this volume was reviewed in this Bulletin in vol. 31 (1925), pp. 566–67. The second edition differs from the first only in unessential details, mainly matters of rewording in a number of places. One error which crept into the first edition might well have been corrected in the second; viz., on p. 116 the author states that the equations $x-e^x\cos y=0$, $y-e^x\sin y=0$ have but one common solution. By sketching the curves it is easily evident that they have an infinity of solutions, which fact one might easily guess by considering the equivalent equation $e^z=z$ in the complex variable.

T. H. HILDEBRANDT