SHORTER NOTICES

Dynamik. By Dr. Wilhelm Müller. Berlin and Leipzig, Walter de Gruyter, 1925. Part I, *Dynamik des Einzelkörpers*, 160 pp. Part II, *Dynamik von Körpersystemen*, 137 pp.

Précis de Mécanique Rationnelle. By Georges Bouligand. Vol. I. Paris, Librairie Vuibert, 1925. viii+282 pp.

Introduction Géométrique à la Mécanique Rationnelle. By Charles Cailler. Ouvrage publié par H. Fehr et R. Wavre. Genève, George et Cie., Paris Gauthier-Villars, 1924. ix+627 pp.

The two small volumes by Müller follow the aim of the Göschen series to give with brevity the essentials of a subject. The evident purpose has been to fit these two books to the technical student as well as the mathematical student who is desirous of becoming acquainted with dynamics as an application of his science. Thus each chapter opens with a theoretical discussion and closes with a problem as practical as possible. Vectors are introduced immediately and used everywhere to add to the compactness of the treatment.

The first chapter treats of the motion of a point in terms of velocity and acceleration ending with curvilinear motion in space. In the second chapter, mass and force are added and Newton's laws of motion developed. Motion, involving only translation, in a gravitational field, on a surface, periodic, including both damped and forced vibrations, are typical examples of this chapter. The final chapter studies the general motion of a rigid body thus including rotation. The examples include rolling motion, the engine regulator, the pendulum, closing with the theory of the top.

The second volume of Müller treats of dynamical systems of rigid bodies. In the first chapter the concept of a system is defined, and the various kinds of forces involved are considered, friction in detail on account of its technical importance. The equations of motion are considered for the various portions of a system with such examples as a cylinder rolling on a moving inclined plane, complicated pulley systems, etc.

In the second chapter the general analytic methods are developed drawing on D'Alembert's principle, and the Lagrangian equations are derived. Examples considered are the steam engine with its regulator, the double pendulum, the precession and nutation of the top, and the gyroscopic stabilizer for ships. The third chapter includes the applications of the calculus of variations to dynamics. Much ground is covered well for so small a book.

The *Mécanique Rationnelle* by Bouligand centers its development upon meeting the demands of the French examinations. A perusal of this book will give an idea of the skill in manipulation and the range of topics required of the candidates for the exacting examinations of the "agrégation" and the "licence". The sound advice is given to the student who must be his own instructor to push on to the problems during a first reading, and then with the aid of the knowledge acquired to return to the study of the theory.

The first seven chapters give a rapid introduction into the dynamics of systems. Vectors, motion of a point and of a solid body, center of mass, principles and general theory of dynamics with applications to a solid body, follow in rapid order. The calculus of variations is introduced and illustrated by the geodesics of a surface, and then it is shown how the equations of motion can be considered as extremals of an integral. Constant appeal is made to the theory of Lagrange.

Chapter VIII, the most important in the book, gives a résumé of the theoretical results, and then proceeds to show in detail the methods of dealing with examples which are classified according to the number of the degrees of freedom. Each of the degrees from one to five is illustrated by examples of considerable difficulty, many of which are taken from past examinations. Among the topics of special interest is the use of analysis situs to exhibit the differences of the so-called equivalent dynamical systems. A second volume of this treatise is promised.

After a long period of teaching mathematics and mechanics at the University of Geneva, Charles Cailler retired in 1921. While busy in the preparation of the *Mécanique Rationnelle* he was overtaken by death in 1922. The manuscript in a partly incomplete form was turned over to H. Fehr and R. Wavre for editing. Cailler's object was to give a new presentation of the subjects of kinematics, statics, and dynamics. Fortunately the co-editors were familiar with his point of view, since they were not only his colleagues at the University but also his former pupils. The high esteem in which Cailler was held by all and in particular by the co-editors is expressed with grace in their introduction.

The author makes his treatise depend upon determinants and linear transformations not limiting himself to a euclidean space of three dimensions but assuming a space of n dimensions in which motion without deformation is possible. The book is divided into four parts. The first part is devoted to the laying of this algebraic foundation for which the author claims a double advantage: first that it unifies the whole subject of rational mechanics and second that it makes easier the approach to the relativity theory, a subject which goes beyond the scope of the present volume. The three chapters of this section are, in order: linear forms, quadratic forms, and the theory of linear transformations. These are all developed in detail with constant geometrical interpretation.

The second part treats of the geometry of vectors, forces, and line geometry.

The third part develops kinematics and finite motions with an extended section on quaternions.

The final part is again kinematics and infinitesmal motions. The discussion ranges from simple topics as parabolic motion, harmonic motion, Lissajous' curves to those more advanced, as the transformations of Lagrange, the instantaneous motion of a solid body, the general theory of rolling motion.

The examples and applications are indeed numerous and well chosen throughout the 627 pages and represent the fruits of a life spent in the cultivation of this field. The co-authors consider the correspondence set up between line geometry and the point geometry on a sphere by means of the complex coordinates of a line to be the most interesting and original part of the work.

G. M. CONWELL

Höhere Algebra. By Helmut Hasse. Vol. I, *Lineare Gleichungen.* Sammlung Göschen. Berlin and Leipzig, Walter de Gruyter, 1926. 160 pp.

The author states in the introduction that the fundamental problem of algebra is the development of general methods by which equations, formed by the four elementary calculation operations from the known and unknown elements of a *Körper,* may be solved. This volume treats the solution of linear equations and volume II is to treat equations of higher degree.

In Chapter I the theory of *Körper* and *Integritätsbereiche* is developed, and rational and integral rational functions defined in relation to the elements of *Körper* and *Integritätsbereiche.* Chapter II contains the elements of the abstract theory of groups, developed in a manner analogous to the theory of *Körper.*

Chapters III and IV are devoted to the actual solution of the problem. Chapter III contains a complete solution from the theoretical standpoint, namely, the Toeplitz treatment of linear equations. This method consists essentially in reducing the problem of the solution of a given system of linear equations, satisfying the necessary condition for solution, to that of the solution of an equivalent system of linearly independent equations for which the necessary condition is proved to be sufficient. A vector is defined in this chapter as the coefficients of a linear form, and a matrix as m vectors, each of which is n-membered. Chapter IV differs but little in the material included from the usual chapter on determinants and linear equations contained in elementary texts on the theory of equations. The treatment, owing to the excellent foundations laid in the first two chapters, is most rigorous. It is introduced by a brief treatment of permutation groups.

Notwithstanding the small size of this volume, the author sacrifices nothing of rigor and clarity to compactness. There are several sets of examples in the first two chapters; but, if the reader wishes to apply the theory, he is forced to look elsewhere for problems.

L. T. MOORE