$-u_1u_2'>0$ implies that the derivative $d(u_2/u_1)/dx$ is less than or equal to zero, so that the equation $u_1=0$ can have no root $x_1'>x_2$ preceding the first root x_2' of $u_2=0$. The points 1 and 1′ are therefore surely not separated by 2 and 2′ on the arc E. This is the complete Jacobi condition as described in the reference of the footnote on the preceding page.

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A CONNECTED AND REGULAR POINT SET WHICH CONTAINS NO ARC*

BY R. L. MOORE

A point set is said to be *connected im kleinen*,† or *regular*, at the point P if, for every positive number e, there exists a positive number d_e such that if X is any point of M at a distance from P less than d_e then X and P lie together in some connected‡ subset of M of diameter less than e. A point set which is regular (connected im kleinen) at every one of its points is said to be regular (connected im kleinen). The set M is *uniformly* connected im kleinen if for each positive number e there exists a positive number d_e such that every two points of M at a distance apart less than d_e lie in a connected subset of M of diameter less than e. If a point set M is con-

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† Cf. Hans Hahn, *Ueber die allgemeinste ebene Punktmenge, die stetiges Bild einer Strecke ist*, JAHRESBERICHT DER VEREINIGUNG, vol. 23 (1914), pp. 318–322. Also S. Mazurkiewicz, *Sur les lignes de Jordan*, FUNDAMENTA MATHEMATICAE, vol. 1 (1920), pp. 166–209. This conception, as applied to a simple closed curve, was used by Pia Nalli in the paper *Sopra una definizioni di dominio piano limitato da una curva continua, senza punti multipli*, RENDICONTI DI PALERMO, vol. 32 (1911), pp. 391–401.

‡ According to Hahn's formulation, X and P lie in a *closed* and connected subset of M of diameter less than e. It has been customary with me to omit the stipulation that this subset should be closed. However, the set M described below is connected im kleinen according to either definition.

nected, connected im kleinen and *closed*, then every two of its points can* be joined by a simple continuous arc which lies wholly in M. In the present note I will show that there exists a connected and connected im kleinen point set which is not closed and which contains no arc whatsoever.

Let S denote a unit square. For each positive integer n, divide S into n^2 equal squares and let S_n denote this set of n^2 squares. For each square q of the set S_n construct just one closed and connected point set† K_q which contains no arc and which lies wholly within q except that it contains just four points on q, these four points being the mid-points of the four sides of q. For each n let M_n denote the set of points obtained by adding together the point sets K_q for all squares q of the set S_n. Let M denote the point set

$$M_1+M_2+M_3+\cdots.$$

This point set is connected and uniformly connected im kleinen. But it contains no arc since it is the sum of a countable number of closed point sets no one of which contains an arc.

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* R. L. Moore, *A theorem concerning continuous curves*, this BULLETIN, vol. 23 (1917), pp. 233–236; S. Mazurkiewicz, loc. cit.; H. Tietze, *Ueber stetige Kurven, Jordansche Kurvenbogen und geschlossene Jordansche Kurven*, MATHEMATISCHE ZEITSCHRIFT, vol. 5 (1919), pp. 284–291.

† That such a set exists may be easily seen with the help of an example described by B. Knaster in his paper *Un continu dont tout sous-continu est indécomposable*, FUNDAMENTA MATHEMATICAE, vol. 3 (1922), pp. 247–286.