*Einleitung in die Mengenlehre.* 2d edition. By A. Fraenkel. Berlin, Julius Springer, 1923. viii + 251 pp.

In this edition this book appears as volume 9 of the series, *Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen.* The author in writing this edition tried to produce a book which would fulfill the general requirements of that series and also the purpose of the first edition*, i. e. to give an exposition of the elements of Mengenlehre which would be clearly comprehensible to one of little or no mathematical training. The prolixity of exposition necessary in realizing the latter aim hinders the realization of the former. The elegance and, in particular, the conciseness which the initiated desires is missing. However, the author seems to have come as near as possible to fulfilling with a single book both purposes.

The first 151 pages of the new edition contain almost verbatim all of the material of the first 129 pages of the old edition and in addition the introduction of certain concepts, more abstruse than those of the earlier book, and the details of certain proofs which previously were give only in outline. The material of chapter 12, pp. 129–151, of the old edition is elaborated in 90 pages, pp. 151–241, of the 2d edition. These pages are devoted to the foundations of the subject. About half of this space is taken up with a historical sketch of the critical examination of the theory, including an exposition of the paradoxes which gave rise to this critique and a brief description of the various procedures devised to give a logically consistent theory. Here are briefly explained and contrasted intuitionalism, which recently has been the subject of considerable work by Brouwer and Weyl, the method of logicizing as developed by Russell and Whitehead and J. Königs, and the axiomatic method as employed by Zermelo and Hilbert. The remaining half is devoted to a detailed exposition of the axiomatic developement of Mengenlehre which Zermelo produced in the period from 1904–1908 and to a sketch of Hilbert's recent work on the question of a proof of the consistency of a system of postulates. In connection with the Zermelo axiomatic the author gives in fine print an exposition of his contribution to the clarifying of the notion "eine definite Aussage" as used by Zermelo.

The exposition of the Zermelo–Hilbert way of setting up a theory of sets is well done but the brevity with which the other methods, logicization and intuitionalism, are presented makes it impossible to give an exposition of the latter methods on a par with that of the former. This is to be regretted. However, in these 90 pages we have the most readily available means of obtaining an acquaintance with the present state of the theory of the foundations of Mengenlehre

* See this BULLETIN, vol. 27 (1921), pp. 333–334.

and, indeed, of mathematics. Although none of the various theories, the logical theories of Russell and Whitehead and of J. Königs, the intuitional theory, suggested by Kronecker and developed by Brouwer and Weyl, the axiomatic theory brought forth by Zermelo and contributed to by Fraenkel and others and Hilbert's theory of proof, which are described here, is in a finished form, they all seem to lead to viewing mathematics and logic as a game in which the pieces and the rules of operation are defined as concretely as possible.

The book closes with a chapter of a few pages which contains a discussion of the connection between Mengenlehre and other branches of mathematics and a bibliography of the subject. These last pages are nearly identical with the corresponding ones of the first edition.

No errors in definitions or proofs have been noted, and the typography is excellent, as in previous volumes of the Springer series.

G. A. PFEIFFER

*Theorie der Differentialgleichungen. Vorlesungen aus dem Gesamtgebiet der gewöhnlichen und der partiellen Differentialgleichungen.* By Ludwig Bieberbach. Berlin, Julius Springer. 1923. viii + 317 pp.

"To write a text book on differential equations which shall be a suitable textbook, and at the same time set forth the spirit, methods and results of the theory in all of their aspects, so that the student shall be prepared to read original papers intelligently—that appears to be an impossibility." With these words Bieberbach opens the preface to the book under review, which is intended only as an introduction to the subject. Its scope is best brought out by giving the titles of the four sections into which the book is divided.

Section I. Ordinary differential equations of the first order.

Section II. Ordinary differential equations of the second order.

Section III. Partial differential equations of the first order.

Section IV. Partial differential equations of the second order.

It is limited therefore to equations of the first and second order.

The author is much more interested in bringing out the meaning and the interpretation of differential equations than he is in setting exercises for the student; in fact, there are no exercises at all. Existence proofs are given, and discussions of the nature of the integral curves. He does not limit himself to real values of the variables, for in each of the first two sections he has a chapter dealing with the the complex values of the variable and the nature of singularities.

While the book is elementary in character, its spirit is scholarly, and the book is well worth while to the student who has not gone deeply into the subject of differential equations.

W. D. MACMILLAN