ON THE ACCESSIBILITY OF AN ARC FROM ITS COMPLEMENT IN SPACE OF THREE DIMENSIONS

(Extract from a letter to R. L. Moore, dated Jan. 29, 1924)

BY CASIMIR KURATOWSKI

In a paper on continuous curves printed in the BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, July, 1923, you have raised the question, *whether an arc lying in a three-dimensional space is accessible at each of its ends from every point which does not lie on it.*

I shall answer *positively* this question.

Let AB be an arc lying in the space S, X a point of $S-AB$, P a point of AB (not necessarily an end-point). Let H be a plane containing the points X and P.

The point-set G, composed of those points of the arc AB which lie on H, is a closed plane and bounded set. Since the arc AB passes through G, the set G does obviously satisfy a condition (which you gave with Professor Kline in a paper printed in ANNALS OF MATHEMATICS)* which is necessary and sufficient in order that it should be possible to pass an arc through it. Let T be an arc lying on H, containing G but not X. Hence T is accessible at each of its points on the plane H (see, e. g., Schoenflies). Let XP be an arc lying on H and having in common with T only the point P. Hence XP has in common with AB only the point P and thus the theorem is proved.

Obviously the proof holds true in n-dimensional space, $n \geqq 3$.

THE UNIVERSITY OF WARSAW

* The paper here referred to is *On the most general plane closed point-set through which it is possible to pass a simple continuous arc*, ANNALS OF MATHEMATICS, (2), vol. 20 (1919), pp. 218–223. R. L. M.