was neither one of the "earliest" nor yet one of the "greatest mathematicians" of England.

In spite of the introduction which should have been prepared by one more familiar with the history of mathematics the work is highly to be commended, as eminently worthy a place in mathematical libraries.

Invitation is extended to subscribe to a guarantee fund to cover the cost of plates for Part III, Astronomy. American libraries are urged to send subscriptions to Part III to R. T. Gunther at Magdalen College, Oxford.

L. C. KARPINSKI

Geschichte der Elementar-Mathematik in systematischer Darstellung. By Johannes Tropfke. Berlin and Leipzig, Vereinigung Wissenschaftlicher Verleger. Bd. I: *Rechnen.* 1921. vi + 177 pp. Bd. II: *Allgemeine Arithmetik.* 1921. 221 pp. Bd. III: *Proportionen. Gleichungen.* 1922. 151 pp. Bd. IV: *Ebene Geometrie.* 1923. 238 pp.

The first edition of Tropfke's *Geschichte der Elementar-Mathematik* was published in 1902–1903, and was reviewed by J. W. A. Young in this BULLETIN.* The four volumes under review form part of a second edition, which is to include seven volumes, the last three being yet in press, or at least not yet available in this country. That the revision has been thoroughgoing is evidenced by the fact that the material corresponding to these four volumes occupied approximately 510 pages in the first edition, and has thus been expanded above 50 per cent; while the references to the literature in these four volumes number 4348, as compared with 1951 in the corresponding parts of the first edition.

And the advance made beyond the first edition is by no means merely quantitative. The author was fortunate in having the active assistance of G. Eneström and H. Wieleitner in the preparation of the new edition, and their names are a sufficient guarantee that no pains have been spared to make the work as complete and authoritative as possible. On nearly every page we find valuable additions to the information given in the first edition; while in several cases the point of view then adopted has been radically changed or even reversed.

To mention but a few of the changes: (1) In discussing the origins of the number system, use has been made of recent researches as to the knowledge of the Babylonians. An interesting detail is the fact (I, p. 15) that as early as 2500 B. C. the tables of Senkereh contain representations of very large numbers according to the sexagesimal system, the largest to date deciphered being $60^8 + 10 \cdot 60^7$ ($= 195\,955\,200\,000\,000$). (2) The history of the development of technical terms is very largely expanded. (3) The independence of the work of the Hindus is in many cases questioned or denied in the second edition where it was accepted in the first; the researches of G. R. Kaye are largely responsible for the author's change of view here. (4) The account of complex numbers (II, 79–90) is entirely rewritten and considerably enlarged. (5) The discussion of the development of the theory of parallels (IV, 53–60) is a considerable improvement over that in the first edition, while it must be confessed that it still leaves something to be de-

* Vol. 12 (1905), pp. 138–140.

sired; thus Saccheri is only referred to in a footnote, and Lambert is not mentioned at all. (6) The work of several individuals has evidently been more thoroughly investigated; thus Leonardo of Pisa's monumental work is more adequately described than in the first edition, while at the same time further information is given as to his Arabian sources. Apparently, however, Abu Kamil has been overlooked among these.* (7) The discussion of indeterminate equations is expanded from 10 to 15 pages (III, 99–114). Here, as well as under the topic "properties of integers" (I, 93–110), we miss any reference to Dickson's *History of the Theory of Numbers*, which would have been of great value to the author, but which presumably was not available during the work of revision.

Two topics which the reviewer would have been glad to see included but which do not seem to be mentioned are the "regula falsi," and the so-called "Russian peasant method" of multiplication. A temporary drawback is the lack of an index, as that is to appear in the last volume. This is of itself a sufficient reason why the publication of the remainder of the work will be awaited with impatience.

A few minor slips or misprints were noted: In referring to the use of the word "cipher" for "zero" (I, 9–10) the author implies that this use is obsolete in English. The date of Leonardo of Pisa's birth is not known to be 1180 (III, 121). The credit for the proof of Steiner's Malfatti problem construction is given to Schroeter, 1874 (IV, 126), whereas it should be given to A. S. Hart, who published a proof in the QUARTERLY JOURNAL OF MATHEMATICS in 1857 (vol. I, pp. 219–221). Misprints occur several times as to dates; thus Albert Girard died in 1632, and the date is so printed in volume I, page 7, but thereafter 1633 is several times found. (II, 76, III, 95, and IV, 123, for example.) Roger Cotes was born in 1682, instead of 1652, as stated on page 159 of volume II. The correct date is given at page 213 of the same volume. There is a misprint on page 134 of volume II, referring to the "fragments of Kahun," which owing to the lack of an index is not easy to correct. The first reference in the "Zeittafel," volume III, page 115, should be I, 128, instead of I, 24. (I, 24 is correct if referring to Leonardo of Pisa's *Scritti*.)

It would be desirable that a reference work of this sort, so near to complete accuracy, should be made even more valuable and kept up to date by the publication at intervals of additions and corrections which might be contributed by anyone interested; possibly the AMERICAN MATHEMATICAL MONTHLY might serve as the medium for such publication.

Tropfke's style is clear and lucid, enlivened by imagery where this can be used without detracting from accuracy. Biographical details are entirely omitted; had they not been, the size of the work must have been expanded to impossible proportions. The topical arrangement brings advantages which fully compensate for the lack of unity which it necessitates. From every point of view the revised edition must be pronounced a decided success, so far as the four volumes permit a judgment of the whole. It can be recommended most warmly to everyone who is interested in the history of elementary mathematics.

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* Cf. L. C. Karpinski, *The Algebra of Abu Kamil*, AMERICAN MATHEMATICAL MONTHLY, vol. 21 (1914), pp. 37–48.