Ballistik. By Dr. Theodor Vahlen. Berlin, Vereinigung wissenschaftlicher Verleger, 1922. xi + 226 pp.

This book is an outgrowth of interest and investigation that were stimulated in the subject of ballistics during the World War. The author, who is professor of pure and applied mathematics in the University of Greifswald, served as an artillery officer, and during that period contributed a number of articles on ballistics to scientific and to military journals. These articles are referred to in the beginning of the book where a rather extensive bibliography on the subject is given, from the work of John Bernoulli in 1719 to the works of Cranz and of Lorenz in 1917.

The reason for writing the book is that there is no book in German literature which presents the mathematical developments of the subject in detail. Cranz has deliberately avoided these in his compendious work which is indispensable to the practical ballistician because of its useful tables and numerous illustrative examples. As no real progress in the science of ballistics is possible without a very close coordination between the theoretical and the practical, the author aims to supply a definite demand by presenting the subject from the standpoint of the mathematician.

There are four divisions of the subject: exterior ballistics, the behavior of the projectile after the powder gas has exerted its entire accelerating effect; interior ballistics, the behavior of the projectile until it begins to leave the muzzle; transition (Übergangs) ballistics which embraces the interval between exterior and interior; and, *Endballistik* which includes the functioning of shells, recoil, etc. The subject is divided further into old ballistics which rests upon the assumptions that the earth is flat, motionless, that gravity, both in magnitude and direction, and the density of the air (upon which the resistance of the air depends) are constant throughout the trajectory, and modern ballistics in which greater ranges and altitudes are encountered and in which the former assumptions are no longer adequate. The claim is made that both transition ballistics and, what I have called *modern* ballistics are treated systematically for the first time in this book. The author seems disposed to name the latter *universal* (that is, as applying to the universe instead of to the earth only) ballistics but refrains from doing so in that sense because the motion of a meteor belongs to the field of celestial mechanics and cannot be regarded a problem of what he calls "kosmische" ballistics.

Under the head of modern ballistics he gives just about what has been developed in this country during the same period, both in regard to the trajectory itself and in regard to small corrections. If there is any difference, his mathematics is more elementary. The coordinates of a point on the trajectory are expressed as power series in t (the time); in this country the coordinates of a point on the trajectory are expressed as functions of velocity, acceleration, and a quantity E which depends upon velocity, altitude, and the ballistic coefficient. The two methods are theoretically the same if we admit that our x and y may be expressed as power series in t. If different results are obtained by the two methods, it would be due to inaccuracies in tabulated physical data.

As the author has so strongly stressed the necessity of developing the mathematical and the empirical aspects of the science side-by-side as a condition precedent to making any permanent progress, he is open to criticism on exactly those grounds in his treatment of correcting for curvature of the earth. To show that this correction is of practical value he takes the extraordinarily long range of 112.8 km. (about 70 miles) and points out that in this case the correction would be 1 km. "Surely such a quantity is not to be disregarded." But does it not seem so very large only because we have so little definite knowledge in regard to the probable error of the gun fired at such a range? Would it not have been more interesting to the average reader to know how the correction for a range of 8 or 10 miles compares with the mean deviation of a series of rounds fired at these more usual ranges? Also it might be well to include a numerical example taking the gun and the target at different heights above sea-level, especially since this could be done by using the formula which is given. The same sort of criticism might be made in regard to the corrections for rain and for an increase in the temperature of the gun due to firing. Experience in this country would seem to indicate that rain would have to be further classified into mist, heavy rain, etc.

In interior ballistics the author follows Sarru and Charbonnier. The three principal formulas are as follows: (I) connecting pressure and density of loading; (II) connecting the linear velocity of the burning of the powder with density and pressure; (III) the work done in the adiabatic expansion of a gas from a finite to an infinite volume. Other familiar formulas derived in this section are those relating to variation of pressure in the barrel, the maximum pressure, and the muzzle velocity.

Although the author emphasizes the importance of transition ballistics and the fact that he presents the subject systematically for the first time, he adds nothing to the subject which is not already very familiar to those interested in the subject. The projectile does not attain its maximum velocity at the muzzle but it is accelerated by the powder gas for some little time afterward. It is also during this interval that the projectile is given a yaw the effect of which, both upon range and upon deflection, is much greater than was formerly supposed. This is one of the problems that remains to be solved and it constitutes one of the most interesting and most advanced fields of investigation in ballistics. The only contribution is a definite name for this important division of the subject.

The later chapters contain a mathematical discussion of probability as related to problems in artillery and suggestions regarding indirect fire.

The author has been successful in making his book mathematical, which was one of his aims. In the reviewer's opinion he has not developed the subject as if the mathematical problems were suggested by observations made in practical experimentation. He pointed out the necessity for this and lamented the fact that it had not been done consistently in the past. The book will be of considerable interest to the mathematician who wishes to acquaint himself with the subject of ballistics; it will be of little value to the practical artillery officer or to the investigator in practical ballistics.

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